Inertial Forces as the Internal Motion of Extended Bodies

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Abstract

I re-examine the nature of inertial frames of reference in connection with the concept of motion embedded in Newton's laws, distinguishing between the translational motion of point masses and the internal motion of extended bodies: translational motion is defined in terms of changes in relative position, whereas internal motion is defined in terms of changes in shape and orientation. By re-examining classic thought experiments from the history of physics that purport to demonstrate the existence of inertial forces, I show that these inertial forces describe precisely the tension and compression internal to extended bodies. I conclude that the defining feature of models with an inertial frame of reference is not actually the motion of the frame of reference, but rather their consistent description of point particles in translational motion, which are the only objects to which the laws of mechanics apply.

1 INTRODUCTION

The basic laws of mechanics are widely understood to be valid only in inertial reference frames, the set of reference frames that are related by Galilean transformations (Møller [1952] 1955, 4; Friedman [1983] 2014, 13). In accelerating, non-inertial reference frames, models of mechanics require the inclusion of 'fictitous' inertial forces in order to describe correctly the behavior of bodies (Feynman et al. [1963] 2010, Ch. 12-5; Sciama 1969, 7). But there is active debate as to the very meaning of the 'motion of a reference frame', which seems to require some notion of absolute space (DiSalle 1990, 139–40). Absolute space presents a philosophical problem (Norton 2019) according to the epistemological principles of Descartes, Leibniz, Berkeley and Mach, who generally disclaim the inclusion of unobservable entities in physical theories (Rynasiewicz 2000). Contemporary analyses of the nature of inertial frames of reference then often focus on the nature of space-time itself, in particular as codified by the general theory of relativity, and on the ontological status of the referents of general relativity (e.g. Earman (1989) and Stevens (2020)).

These issues are especially relevant in light of recent work by Saunders (2013) showing that the concept of an inertial frame is not in fact integral to Newtonian mechanics. Saunders develops a space-time representation of Newton's laws that encodes only the precise dynamical symmetries of the classical theory, taking into account Corollary VI to the Laws of Motion, and demonstrates that linear acceleration, if not rotation, is locally undetectable. I would like to add to this discussion by directing it away from the development of space-time formalisms and toward a conceptual analysis of the dynamical thought experiments that provide us with our basic understanding of the nature of motion from classical mechanics. In doing so, I will side-step the ongoing debate concerning the reality of the objects to which the laws of mechanics refer and instead examine the "semantical structure" of the laws themselves (Hanson 1963, 110). That is, I would like to reframe the question of *what* it is these laws are referring to when they describe the dynamics of bodies, as a question of *how* they do it at all. I believe that this approach should provide some clarity in particular as to why rotation must be treated as a special case in the work of Saunders and others.

I begin by reviving an old distinction between two different kinds of motion: 1) motion that is a *change of place* of a body, and 2) motion that is *internal to that body* (for example, rotation). There is long tradition of natural philosophers treating motion as being of one of these two types (cf. Copernicus (1543, 6) and Kant ([1884] 1993, 45)). After Newton, however, the historical focus was usually exclusively the former (Newton et al. 1962, 126–27), and with the development of analytical mechanics the distinction between translational and internal motion completely disappeared; efforts to formalize the science refer explicitly to the motion of point particles or aggregates thereof (Pinheiro 2011, 1; DiSalle 2020). Notably, in the context of contemporary efforts to establish the fundamental principles of mechanics, there is no great difference between substantivalists and relationists in this regard, as recognized by Rynasiewicz (2000, 299–303): relationists naturally attempt to model all physical dynamics in terms of distances between point particles (Vassallo et al. 2017, 4), and substantivalists rely on some notion of absolute space with an accompanying metric, the defining feature of which is this same relation (e.g. Maudlin 2007, 87–89).

When we then revisit the thought experiments that have historically been used to demonstrate the existence of inertial forces in accelerating frames of reference, we see that these 'pseudo-forces' are all forces of the *tension* and *compression* of extended bodies, rather than the acceleration of point particles. Whereas it is widely taught that the presence of such forces is a consequence of the accelerative motion of the reference frame for the measurement, I argue that if we apply Newton's laws only to point particles in these thought experiments, then the pseudo-forces disappear. The motion of the reference frame itself is not its defining characteristic: a model with an inertial reference frame should be understood not as one that is moving in a certain way, but rather as one that describes the motion of bodies exclusively as the *translation of point particles*. A model with an 'inertial frame of reference' may therefore be understood as simply as any model in which all motion is the translational motion of point masses, there are no extended bodies in internal motion, and all of the forces are balanced by Newton's Third Law.

I am not proposing any alternative to, or extension of, a theory of mechanics, for instance one that might explain the tension in the Rotating Spheres thought experiment as arising from the (counter-)rotation of the distant stars, *per* Mach (1921, 284); I am simply attempting to describe a limitation of mechanical pictures of dynamical motion that are based around descriptions of point masses in translational motion, which pictures become problematic when they are also applied to internal motion. I conclude that we should *expect* theories from this tradition to be unable to describe the internal motion of extensive bodies, and we should understand our concepts of inertial frames of reference and pseudo-forces as arising from this defining feature of classical mechanics.

2 TRANSLATIONAL AND INTERNAL MOTION

Definition 1. A **simple body**, **point particle** or **point mass** is an object occupying a single point in space at any given time, with no properties other than a mass, m. (A simple body has no extension, shape, or orientation.)

Definition 2. An **extended body**, **composite body** or **complex body** (of mass m) is an object with extension, shape and orientation in space. Unlike a simple body, an extended body does have 'parts' and 'sides' in some definite relation to each other.

Definition 3. **Translation** is the motion of a body through space. It is well-defined only for point particles in relation to other point particles.

Definition 4. **Internal motion** is a change in the size, shape, or orientation of an extended body, such as rotation, compression or expansion.

If I say that an object is moving, I could mean that it is moving translationally, like an object riding on a conveyor belt; or I could mean that it is moving 'internally', like a body in rotation or a working machine. In the former case, I am describing the object as a simple body, with no component parts and no internal motion. In the latter case, I am treating the body implicitly as comprising multiple parts in some arrangement (the sides of a spinning object, or the gears of a clock, for example). On one hand, it obviously doesn't mean anything to talk about the internal motion (e.g. the rotation) of a point particle. On the other hand, the translational motion of an extended body—even its position—is also undefined: My measurement of the position of a grandfather clock, say, depends on which part of the clock I am pointing my laser rangefinder at, as it were.

For there to be only one answer, I should have to treat the clock as occupying a single point in space, calling any differences in the possible measurements insignificant (Kant [1786] 1787, 5). Similarly, if I were to try to measure the relative, translational motion of a body that is itself moving internally—say when pointing a Doppler radar gun at a spinning beach ball—then the result would also depend on what part of the beach ball I pointed the radar gun at. In order to be able to talk about the translational motion of a body toward or away from me, I should have to model it as internally at rest, i.e. as rigid and non-rotating (Boltzmann 1974, 142; Hertz [1894] 1899, 223–24).

We may model each kind of motion as motion of the other kind: We may describe a spinning beach ball either in terms of the rotation of the (extended) ball itself, or in terms of the translational motion of each of the *parts* of the beach ball *as simple bodies* (with no parts themselves) as they each travel in circles around the axis of rotation. Equivalently, the translation of an object along a conveyor belt may be described also as the internal motion of some (extended) assembly line comprising both the object and the conveyor belt, where the assembly line itself isn't moving at all (cf. Husserl [1934] 1981, 227) and it is merely the internal arrangement of the assembly line *qua* extended body that then changes. The difference is whether we are treating the parts of a body as the parts of some extended body, or instead as independent, simple objects in and of themselves.

The critical point is this: *in mechanics, there are no descriptions of internal motion*. All motion is represented as the translational motion of point masses through space, as it is only for point masses that positions, distances and translations are even well-defined; what might be described as an extended body is always represented as a mere collection of point particles in translation (Boltzmann 1974, 143, 224; Hertz [1894] 1899, 126). So Descartes defined motion as "the transfer of one piece of matter, or one body [...] to the vicinity of other bodies" (Descartes [1644] 1897, 53), and Newton wrote, "[...] all these motions of the wheels of the clock [...] are truly and philosophically speaking in the particles of the wheels" (1962, 126–27). In physics, when we talk about the translational motion of a clock, we are always really talking about the motion of its center of mass (at a single point in space), and when we talk about the rotation of an extended body, that motion is understood as the *cyclical* (e.g. circular or elliptical) motion of the parts of the whole as simple bodies themselves—that is, as the translation of those simple bodies, where the direction of their motion is constantly changing (Galilei [1590] 1960, 64; Berkeley [1721] 1992, §61).

3 Force and Motion

Just as we can talk about both translational and internal motion, so are there forces of both translational and internal motion. But mechanics treats only forces of the former type, never e.g. the contraction or expansion of extended bodies. Accordingly, two different ways of talking about force are as follows: 1) We can talk about the force acting on a hockey puck as I push it across on an ice rink. This force manifests as a change in the motion of the puck as a whole—as the acceleration of its center of mass; 2) we can talk also about the forces acting on a spring that I am squeezing between my fingers. In this latter case, the spring is compressed, but does not itself move at all—its motion is entirely internal. Of course, it is possible to describe the compression of the spring in terms of the motion of the two ends of the spring, rather than as a change in the shape of the spring itself. But our models from mechanics treat every body specifically as some set of (incompressible) point masses in space with independent displacements.

Within mechanics, a force is *defined* as a change in translational motion of point particles, and Newton's Second Law identifies all 'forces' with the accelerations of centers of mass ([1687] 1934, 83).

Drawing this distinction between translational and internal motion—and between forces of translational and internal motion—allows us to re-examine the thought experiment of a passenger feeling an inertial force in an accelerating train (Born [1924] 1962, 77). This thought experiment is often used to illustrate the existence of inertial forces in non-inertial reference frames; but when we carefully distinguish between the translational and internal motion of the bodies in the model, we see that the inertial forces never appear.

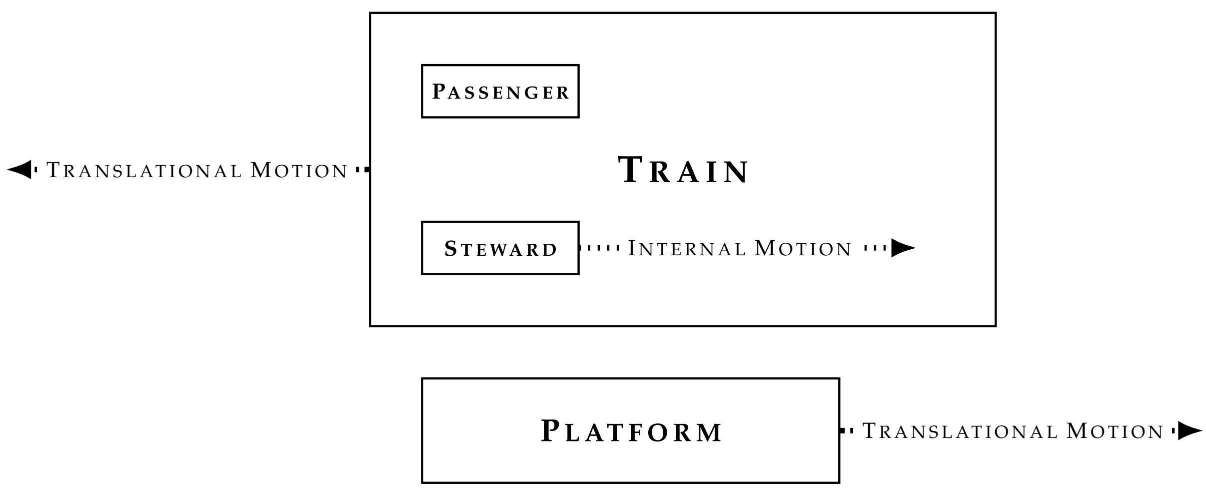

Figure 1: Inertial forces are internal motion of a extended body that is itself a part of another moving extended body.

Consider a train accelerating from a stop, with a passenger sitting in a seat facing forward and also a steward on roller skates standing in the aisle (Figure 1). As the train moves forward, the steward rolls backward in the train and the passenger is compressed by the chair. What kind of force does the steward's body feel, as compared with that of the passenger? The steward's body is accelerated backward relative to other bodies in the train, but is not compressed; the passenger's body is compressed, but is not accelerated backward: The passenger's body is fixed in his seat, and it is only insofar as this is the case that the passenger feels the compression. The steward moves as a simple body, in translation; but it is only the *parts* of the passenger's body (i.e. their internal organs) that move at all (in the frame of the train). The effect on the steward is purely translational; the effect on the passenger is purely internal.

Which of these two bodies is feeling an inertial force? By Newton's Second Law, the passenger is not feeling any force at all, because they are not accelerating. On the other hand, the steward is accelerating backward in the frame of the train, but in exactly the same way as the 'stationary' platform outside of the train. Would we say that the train platform—stationary relative to the Earth—is feeling an inertial force pulling it backwards as the train accelerates? The only difference between the steward and the platform is that the steward is inside the train and the platform is not. So the inertial force that is normally identified in this thought experiment exists as acceleration only *inside* a body (the passenger) that is itself *inside* a forward-accelerating body (the train). That is, the compressive force acting on the passenger is the motion of the body parts of the passenger, rather than the motion of the passenger themself. If we restrict ourselves to identifying forces only where there is accelerative motion, and if we consistently model all of the elements of this system as point particles, then there are no longer any inertial forces in a model of the above system. The steward on roller skates is not 'inside' the train—which is a mere collection of point masses—any more than the platform or a bird flying overhead is: they are simply not a part of the model, and indeed their motion would violate Newton's Third Law, Conservation of Energy and Conservation of Momentum.

4 Non-Inertial Frames of Reference

In an accelerating frame of reference, even point particles that would otherwise be at rest will then themselves accelerate, and by a naïve application of Newton's Second Law, feel a fictitious force. However, as above, this kind of 'force' is not properly the result of *inertia*, and such a 'force' is always locally undetectable, unlike say the compression of the train passenger against their seat. Newton himself noted in his VIth Corollary to the Laws of Motion that a system accelerating along parallel lines is locally indistinguishable from one that is not accelerating ([1687] 1934, 89). That is, when a system is moving *as a unit*, even if its motion is accelerative, that motion requires the introduction of no inertial forces to the model, and so it is not the acceleration of a reference frame *per se* that distinguishes models with inertial forces from those without, but rather which elements are actually a part of the system.

Rotation, nevertheless, seems to be a special case. Indeed, the classic argument for the special status of inertial frames of reference within mechanics is based on Newton's "Rotating Spheres" thought experiment ([1687] 1934, 82): for two spheres connected by a string and rotating around the common center of mass, there is tension in the string connecting the spheres ('the centripetal force'), and, by the Third Law of Motion, every force must have an equal and opposing force, which is the pseudo-force called 'centrifugal'.

The problematic nature of rotation is elucidated by recognizing the above distinctions between internal and translational motion, and in particular the distinction between internal rotation and cyclical translation: in the rotating frame, the spheres are not moving translationally; by the Second Law then, there is no force acting on them. We introduce the centrifugal force in order to balance the force of tension in the string—an extended body—but we are still treating the spheres as point particles (e.g. Newburgh 2007, 427–28). If we treat both the string and spheres as point masses with no internal motion, then neither feels any forces whatsoever in the Newtonian sense. And if we treat *both* the string and the spheres as extended bodies, then the tension in the string balances the tension in the spheres, and the sum of the accelerative forces is again zero, without any centrifugal forces.

The trick is that, when we specify 'in the [non-inertial] frame of the rotating body' (Feynman et al. [1963] 2010, Ch. 19-4), we are implicitly doing more than making a mere coördinate transformation, i.e. a change in position, velocity, acceleration, etc. of the frame of reference, to somewhere on the axis of rotation, and so forth; we are shifting from a model of the circular translation of the simple parts of the rotating body *qua* collection of point masses to a model that also describes the internal forces of tension, etc. of the rotating body *qua* an extended object. It is this change that introduces the inertial forces. And this is why the Newton-Huygens space-time of Saunders, from his analysis of Newtonian mechanics, is able to have no notion of absolute acceleration, and yet still must distinguish between different absolute orientations of bodies (2013, 20–21): extended bodies have orientations but no well-defined relative displacements, while point particles are the opposite.

It is precisely the same sort of conflation between internal and translational motion that is responsible for the qualified applicability of 'quasi-inertial frames of reference' described in Newton's VIth Corollary. These frames are considered to be merely "approximately" inertial (DiSalle 2020) because of, in particular, the tidal forces that would be present in a realistic gravitational field (Schutz 1990, 125). Of course, these tidal forces are nothing but the forces of compression acting on extended bodies in both the Newtonian and Einsteinian theories of gravity (Knox 2014). Indeed, the Equivalence Principle itself applies exclusively to point particles, for only with point particles are there no tidal forces. So within general relativity, it is precisely models of point particles that describe inertial motion (Wald 1984, 73–74), which is motion along a geodesic on the space-time manifold.

I conclude that we should continue to re-evaluate the utility of the concept of an 'inertial frame of reference', which has no place in Newton's original theory of mechanics and was developed no less than two centuries after his introduction of the fundamental laws of motion (Lange 1885, 273). We have seen that in two of the primary thought experiments that purport to demonstrate the existence of inertial forces, those inertial forces are found precisely in the compression or expansion of extended bodies, to which Newton's laws cannot be applied directly. Instead of attempting to classify reference frames as inertial or non-inertial, we might be better served simply by recognizing that the laws of mechanics (as well as those of general relativity) provide a picture of the world that describes precisely the translational motion of point particles in a closed system (where the forces are all balanced). Therefore, a model to which the First Law applies is one whose elements are exclusively these infinitesimal masses, as necessitated by the definitions of distance and relative displacement on which our mechanics is based.

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