

A GRAMMATICAL INVESTIGATION INTO THE  
CONCEPTS OF WESTERN PHYSICS, AND  
INFINITE SPACE AS FAUSTIAN PRIME SYMBOL

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*Eine Ehrenrettung der Scholastik steht, wenn nicht alles täuscht, auch in dieser Beziehung bevor, sie ist schon im vollen Gange. Kopernikus wird von Ptolemäus geschlagen werden. Die heliozentrische These begegnet nachgerade einem geistigen Widerstand, dessen Unternehmungen wahrscheinlich zum Ziele führen werden. Die Wissenschaft wird sich philosophisch genötigt sehen, die Erde in alle Würden wieder einzusetzen, die das kirchliche Dogma ihr wahren wollte.*

—Thomas Mann<sup>1</sup>

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<sup>1</sup> Mann [1924] 1952, p. 503 [Translation: “The honor of the scholastics will be vindicated in this regard as well, if I am not mistaken. Indeed the process is well under way. Copernicus will be routed by Ptolemy. The theory of heliocentrism is now being opposed by intellectual forces whose efforts will presumably attain their desired goal. Science will find itself philosophically constrained once again to grant earth all the honors that Church dogma wished to preserve for it.” ([1924] 2005, Ch. 6, The City of God and Evil Deliverance)]

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## INTRODUCTION

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The aim of this work is to serve as an investigation into the logic, or grammar,<sup>1</sup> of the words and concepts relating to the natural world, for instance ‘motion,’ ‘determinism,’ ‘observation,’ and ‘space,’ the same way one might examine the grammar of expressions of ‘pain’ in the philosophy of mind<sup>2</sup> or the significance of ‘Romanticism’ in the philosophy of history<sup>3</sup>. The practice of natural philosophy, in contrast to that of Natural Science, explicitly does not make use of theoretical or empirical methods of any kind. Like other branches of philosophy, the philosophy of nature is concerned with what is *necessary* rather than what is accidental (i.e., incidental); it is concerned not with constructing ever-more sophisticated models for predicting the behavior of particular phenomena, but rather with improving our understanding of the fundamental, foundational concepts of our world-view for its own sake.

For there seemed to pertain to logic a peculiar depth—a universal significance. Logic lay, it seemed, at the bottom of all the sciences.—For logical investigation explores the nature of all things. It seeks to see to the bottom of things and is not meant to concern itself whether what actually happens is this or that.—It arises, not from an interest in the facts of nature, nor from a need to grasp causal connexions: but from an urge to understand the basis, or essence, of everything empirical. Not, however, as if to this end we had to hunt out new facts; it is, rather, of the essence of our investigation that we do not seek to learn anything new by it. We want to understand something that is already in plain view. For this is what we seem in

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<sup>1</sup> “*Essence* is expressed by grammar.” —Ludwig Wittgenstein ([1953] 1958, §371)

<sup>2</sup> E.g., “Our language employs the phrases ‘my pain’ and ‘his pain’ and also ‘I have (or feel) a pain’, but ‘I feel my pain’ or ‘I feel his pain’ is nonsense.” —Ludwig Wittgenstein ([1964] 1975, §65)

<sup>3</sup> Spengler [1918] 1926a, Ch. III: The Problem of World-History, I. Physiognomic and Systematic, §VII, p. 108.

some sense not to understand.

—Ludwig Wittgenstein<sup>4</sup>

Today, all modes of inquiry into the natural world that are not based, explicitly or implicitly, on the methods of Natural Science are understood to be patently anachronistic and ultimately valueless. The very existence of philosophy proper seems to threaten the scientific progress which we now commonly recognize to be our highest goal and best means of furthering our understanding of the world.<sup>5,6,7</sup> Accordingly, natural philosophy has been effectively dead as an academic discipline for most of the past two centuries,<sup>8</sup> and all efforts in that tradition are now regarded as something between idle fancy and sheer quackery.<sup>9</sup> But Natural Science—and mathematical physics in particular<sup>10</sup>—is a grand edifice built on particular conceptual foundations, and it is within those foundations that many of the greatest problems lie. Questions such as “What is the nature of space?”, “What counts as an observation?”, and “Is the world fundamentally deterministic?”—widely held to fall within the domain of Natural Science<sup>11,12</sup>—are in fact philosophical problems, and they

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4 Wittgenstein [1953] 1958, §89

5 Cf. “The only justification for our concepts and system of concepts is that they serve to represent the complex of our experiences; beyond this they have no legitimacy. I am convinced that the philosophers have had a harmful effect upon the progress of scientific thinking in removing certain fundamental concepts from the domain of empiricism, where they are under our control, to the intangible heights of the *a priori*. For even if it should appear that the universe of ideas cannot be deduced from experience by logical means, but is, in a sense, a creation of the human mind, without which no science is possible, nevertheless this universe of ideas is just as little independent of the nature of our experiences as clothes are of the form of the human body. This is particularly true of our concepts of time and space, which physicists have been obliged by the facts to bring down from the Olympus of the *a priori* in order to adjust them and put them in a serviceable condition.” —Albert Einstein ([1921] 1923, p. 2)

6 “Only rarely did it seem to me [that philosophy of science has] anything to do with the work of science as I knew it. [...] I am not alone in this; I know of *no one* who has participated actively in the advance of physics in the post-war period whose research has been significantly helped by the work of philosophers.” —Steven Weinberg (1993, pp. 133–34)

7 Cf. “Philosophical clarity will have the same effect on the growth of mathematics as sunlight has on the growth of potato shoots. (In a dark cellar they grow yards long.)” —Ludwig Wittgenstein ([1974] 1980b, p. 381)

8 “And then, during the 18<sup>th</sup> and 19<sup>th</sup> centuries, natural philosophy died. It split into empirical science on the one hand, and philosophy on the other.” —Nicholas Maxwell (2012, p. 2)

9 “It has become a tradition among those who talk glibly about science that the romantic *Naturphilosophie* of Schelling and his followers represents the lowest degradation of science and that only by completely freeing themselves from that nightmare were modern biology and medical science able to resume their scientific progress. The incident has been used by empiricists as a moral to warn us against speculative philosophy in the natural sciences.” —Morris Cohen (1947, p. 208)

10 “All science is either physics or stamp collecting. [...]” —Ernest Rutherford (Black 1945, p. 168)

11 Cf. “The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements.” —Einstein, Podolsky, and Rosen (1935, p. 777)

12 “How can we understand the world in which we find ourselves? How does the universe behave? What is the nature of reality? Where did all this come from? Did the universe need a creator? Most

must be treated accordingly.<sup>13</sup> So, General Relativity, for instance, says nothing whatsoever about the nature of space and time;<sup>14</sup> rather, it merely provides a model for the precession of the perihelion of Mercury, the bending of light by massive bodies, etc. ‘Space’ and ‘time’ are concepts that may be used in the description of those predictions; but, notably, all of the same predictions might have been expressed using entirely different concepts.

That is not to say that scientific investigation is useless to philosophy.<sup>15</sup> In fact, the majority of true natural-philosophical thinking in the past two centuries has been performed by scientists (Mach, Hertz, Einstein, etc.) And philosophical thought—that is, *logical* thought—has played a critical role in the development of new physical concepts, in particular within the framework of the *Gedankenexperiment* (thought experiment). Still, the very nature of these thought experiments is, characteristically, widely misunderstood:<sup>16</sup> a thought experiment is described as a kind of experiment “performed in the laboratory of the mind”<sup>17</sup> and based on “empirical data [that are] well-known and generally accepted,”<sup>18</sup> which highlights the intellectual fixation on empirical

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of us do not spend most of our time worrying about these questions, but almost all of us worry about them some of the time.¶ Traditionally these are questions for philosophy, but philosophy is dead. Philosophy has not kept up with modern developments in science, particularly physics. Scientists have become the bearers of the torch of discovery in our quest for knowledge.” —Stephen Hawking and Leonard Mlodinow (2010a, p. 5)

- 13 “If therefore philosophy were to succeed in creating a system such that in all cases mentioned it stood out clearly when a question is not justified so that the drive towards asking it would gradually die away, we should at one stroke have resolved the most obscure riddles and philosophy would become worthy of the name of queen of the sciences.” —Ludwig Boltzmann ([1904] 1974b, p. 167)
- 14 “I had just read Weyl’s book *Space, Time and Matter*, and under its influence was proud to declare that space was simply the field of linear operations.¶ ‘Nonsense,’ said Heisenberg, ‘space is blue and birds fly through it.’” —Felix Bloch (1976, p. 27)
- 15 “Is scientific progress useful to philosophy? Certainly. The realities that are discovered lighten the philosopher’s task, imagining possibilities.” —Ludwig Wittgenstein ([1948–1949] 1982, §807)
- 16 E.g., “At the core of the discussion sits a relatively simple epistemological challenge that is presented in a particularly powerful manner by numerous thought experiments that the history of science has to offer. They suggest that we can learn about the real world by virtue of merely thinking about imagined scenarios. But how can we learn about reality (if we can at all), just by thinking in such a way? Are there really thought experiments that enable us to acquire new knowledge about nature without new empirical data? If so, where does the new information come from if not from contact with the realm of investigation under consideration in an imagined scenario? Finally, how can we distinguish good from bad instances of thought experiments? These questions seem urgent with respect to scientific thought experiments, because many ‘recognize them as an occasionally potent tool for increasing our understanding of nature [...] Historically their role is very close to the double one played by actual laboratory experiments and observations. [...]’” —Stanford Encyclopedia of Philosophy (J. R. Brown and Fehige 2019)
- 17 J. R. Brown [1991] 2011, p. 1.
- 18 Kuhn [1964] 1977, p. 241.

inquiry. Certainly, a thought experiment is nothing more than a logical argument in the form of a hypothetical construction.<sup>19,20</sup>

This work, then, is an attempt to revive natural philosophy as a discipline independent from and complementary to Natural Science. As Galileo described the relativity of motion with his thought experiment of the cabin in the moving ship,<sup>21</sup> so we aim to elucidate the meaning of other such concepts integral to physics, modern and classical. Now, a thought experiment—and philosophical reasoning more generally—certainly says nothing as to whether, for instance, some rock dropped from the mast of a ship will *actually* fall parallel to the mast (because of the wind, say). The goal is not to describe events, but rather to dispel confusion and obfuscation in our thought and language<sup>22</sup> pertaining to (in this case) motion itself: [Galilean] relativity is able neither to be confirmed nor to be refuted by experimentation; but it makes clearer what we mean when we say, for example, “The rock fell *down*.” Ernst Mach, in particular, made great strides in natural philosophy when he attempted to ground the science of mechanics in the analysis of sensations—not because everything is a mere sensation<sup>23</sup> (when I drop a rock on my toe it is not a sensation that has injured me<sup>24</sup>), but because “the meaning of a word is its use in the language.”<sup>25</sup> So the meaning of the word ‘distance’ is seen in how distance is measured (e.g., with a rigid rod, *per* Einstein<sup>26</sup>). We are not concerned with particular facts

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19 “What Mach calls a thought experiment is of course not an experiment at all. At bottom it is a grammatical investigation.” —Ludwig Wittgenstein ([1964] 1975, p. 52)

20 “Thought experiments are arguments.” —John D. Norton (1996, p. 354)

21 “[D]rop a lead ball from the top of the mast of a boat at rest, noting the place where it hits, which is close to the foot of the mast; but if the same ball is dropped from the same place when the boat is moving, it will strike that distance from the foot of the mast which the boat will have run during the time of fall of the lead, and for no other reason than that the natural movement of the ball when set free is in a straight line toward the center of the earth.” —Galileo Galilei ([1632] 1967, p. 126)

22 “Only very slowly and gradually will all these illusions recede and I regard it as a central task of philosophy to give a clear account of the inappropriateness of this overshooting the mark on the part of our thinking habits; and further, in choosing and linking concepts and words, to aim only at the most appropriate expression of the given, irrespective of our inherited habits. Then, gradually, these tangles and contradictions must disappear.” —Ludwig Boltzmann ([1904] 1974b, p. 167)

23 “Bodies do not produce sensations, but complexes of elements (complexes of sensations) make up bodies. If, to the physicist, bodies appear the real, abiding existences, whilst the ‘elements’ are regarded merely as their evanescent, transitory appearance, the physicist forgets, in the assumption of such a view, that all bodies are but thought-symbols for complexes of elements (complexes of sensations).” —Ernst Mach ([1886] 1914, p. 29)

24 “If I wanted to eat an apple, and someone punched me in the stomach, taking away my appetite, then it was [not] this punch that I originally wanted.” —Ludwig Wittgenstein ([1964] 1975, §22)

25 Wittgenstein [1953] 1958, §43.

26 “If, in pursuance of our habit of thought, we now supplement the propositions of Euclidean geometry by the single proposition that two points on a practically rigid body always correspond to the same distance (line-interval), independently of any changes in position to which we may subject the body,

(nor was Mach, though he thought so<sup>27</sup>)—one does not perceive facts about distance itself, but about the distance between such-and-such objects;<sup>28,29</sup> rather, distance is a concept that may be used in the description of the relationship between all objects of a specific form,<sup>30</sup> and the logic of the concept is, *pace* Mach, knowable *a priori*.<sup>31,32</sup>

We are not trying to construct a particular model (or ‘picture’) of the world<sup>33</sup>—which may or may not accord with reality<sup>34</sup>—but rather to describe the model *itself*.<sup>35</sup> My description of the picture is not a description of reality. . . the *picture* is. So when we say, for instance, ‘a[n ideal] billiard table is symmetric in time,’<sup>36</sup> or ‘the three-body problem is chaotic,’<sup>37</sup> we are describing the grammar of those models;<sup>38</sup> we are saying nothing at all about how such-and-such billiard balls or such-and-such a star system will actually behave, which is what the model itself says and that which may be tested experimentally.

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the propositions of Euclidean geometry then resolve themselves into propositions on the possible relative position of practically rigid bodies.” —Albert Einstein ([1917] 1920, p. 3)

- 27 “In the infinite variety of nature many ordinary events occur; while others appear uncommon, perplexing, astonishing, or even contradictory to the ordinary run of things. As long as this is the case we do not possess a well-settled and unitary conception of nature. Thence is imposed the task of [...] seeking out [...] those elements that are the same, and [...] ever present.” —Ernst Mach ([1883] 1919, §5)
- 28 “Time, space, and mass in themselves are in no sense capable of being made the subjects of our experience, but only definite times, space-quantities, and masses.” —Heinrich Hertz ([1894] 1899, p. 139)
- 29 “We ought not to forget that any description of the world by means of mechanics will be of the completely general kind. For example, it will never mention *particular* pointmasses: it will only talk about *any pointmasses whatsoever*.” —Ludwig Wittgenstein ([1921] 1922, §6.3432)
- 30 “Space, time, colour (being coloured) are forms of objects.” —Ludwig Wittgenstein ([1921] 1922, §2.0251)
- 31 “Moreover, when his empirically minded Aristotelian opponent asks him, ‘Did you make an experiment?’ Galileo proudly declares: ‘No, and I do not need it, as without any experience I can affirm that it is so, because it cannot be otherwise.’” —Alexandre Koyré (1943, p. 13)
- 32 “We do not believe *a priori* in a law of conservation, we *know a priori* the possibility of its logical form. [6.33.]¶ All those propositions which are known *a priori*, like the principle of sufficient reason, of continuity in nature, etc., etc., all these are *a priori* insights relating to the possible ways of forming the propositions of natural science. [Cf. 6.34.]” —Ludwig Wittgenstein ([1961] 1979, §23.4.15)
- 33 “The proposition is a model of reality as we imagine it.” —Ludwig Wittgenstein ([1961] 1979, §27.10.14)
- 34 “A system of propositions is laid like a yardstick against reality.” —Ludwig Wittgenstein ([1964] 1975, p. 317)
- 35 “Philosophy is the doctrine of the logical form of scientific propositions (not only of primitive propositions).” —Ludwig Wittgenstein ([1961] 1979, Appendix 1, p. 106)
- 36 See [page 14](#).
- 37 See [page 8](#).
- 38 “Or like saying that a die must fall on one of six sides. When the possibility of a die’s falling on edge is excluded, and not because it is a matter of experience that it falls only on its sides, we have a statement which no experience will refute—a statement of grammar. Whenever we say that something *must* be the case we are using a norm of expression. Hertz said that wherever something did not obey his laws there must be invisible masses to account for it.” —Ludwig Wittgenstein ([1932–1935] 2001, p. 16)



In this work, we attempt to characterize in particular the pictures of the natural world that are found in the **Faustian**<sup>39</sup> (i.e., West-European–American) tradition of science and philosophy. The foundational principles of our physics (Conservation of Energy, the Equivalence Principle, Newton’s Laws of Motion, etc.), commonly described as ‘natural laws’ and as applicable across all physical domains,<sup>40,41</sup> are nothing more than the grammatical features of just these sorts of pictures.<sup>42</sup> These principles, originally established by Galileo and Newton in the development of classical mechanics, found their ultimate expression in Einstein’s theory of General Relativity; but wherever our scientific investigations have led us to physical theories with incompatible grammars (as is the case with quantum mechanics, in particular), there we find philosophical confusion and problems of ‘interpretation.’<sup>43,44</sup> By describing these established pictures, and their limits of applicability, we to hope to arrive at a position to tackle long-standing philosophical problems rooted in persistent misunderstandings of the logic of our world-view and, in the process, to show how the concepts fundamental to that world-view are all connected to each other and to the deep symbols of our Faustian culture.<sup>45</sup>

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39 Spengler [1918] 1923, p. 237.

40 “The workings of our minds and bodies, and of all the animate or inanimate matter of which we have any detailed knowledge, are assumed to be controlled by the same set of fundamental laws, which except under certain extreme conditions we feel we know pretty well.” —P. W. Anderson (1972, p. 393)

41 “First and foremost, the laws are universal [...] The laws are taken to apply unfailingly everywhere in the universe and at all epochs of cosmic history. No exceptions are permitted.” —Paul Davies (1992, pp. 82–83)

42 “Mechanics determines one form of describing the world by saying that all propositions used in the description of the world must be obtained in a given way from a given set of propositions — the axioms of mechanics. It thus supplies the bricks for building the edifice of science, and it says, ‘Any building that you want to erect, whatever it may be, must somehow be constructed with these bricks, and with these alone.’” —Ludwig Wittgenstein ([1921] 1922, §6.341)

43 “The quantum theory, as has been repeatedly emphasized, stands in far greater contrast to classical physics than does the general theory of relativity. The latter, in spite of its fundamental transformation of the concepts of time and space, can be incorporated without great difficulty into the mode of thought of classical physics. Planck says of it that it has brought classical physics to its consummation, as it were, in that by the amalgamation of time and space, it has also united the concepts of mass and energy and those of gravity and inertia in a single higher point of view. Of the quantum theory, however, he remarks that it had the effect of a dangerous foreign explosive which has already caused a gaping rift throughout the entire structure. ‘The introduction of the quantum hypothesis therefore does not represent a modification, as was the case with relativity theory, but a breach in the classical theory.’” —Ernst Cassirer ([1936] 1956, p. 109)

44 “I think I can safely say that nobody understands quantum mechanics [...]” —Richard Feynman ([1965] 1967, p. 129)

45 “But before the curtain falls, there is one more task for the historical Faustian spirit, a task not yet specified, hitherto not even imagined as possible. There has still to be written a *morphology of the exact sciences*, which shall discover how all laws, concepts and theories inwardly hang together as forms and what they have meant as such in the life-course of the Faustian Culture. [...] We shall inquire whence came these forms that were prescribed for the Faustian spirit, why they had to come to our

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kind of humanity particularly and exclusively, and what deep meaning there is in the fact that the numbers that we have won became phenomenal in just this picture-like disguise.”—Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §XV, p. 425)

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 THE GRAND WORLD-MACHINE
 

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*[Western physics], as a science, is an immense system of indices in the form of names and numbers whereby we are enabled to work with Nature as with a machine. As such, it may have an exactly-definable end.*

—Oswald Spengler<sup>1</sup>

In the Faustian conception of Nature, the world is like a great *machine*:<sup>2</sup> underlying all of the “apparent chaos”<sup>3</sup> of our individual perception, there exists the “objective reality”<sup>4,5,6</sup> of an inner mechanism<sup>7</sup> that is eternal, absolute, and perfectly knowable.<sup>8</sup> Our aim in Natural Science is the incremental discovery<sup>9,10</sup> of

*‘Objective  
Reality’*

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- 1 Spengler [1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §I, p. 378
- 2 “My aim is this, to show that the celestial machine is not like a divine creature, but like a clock (and he who believes there is an animate force in a clock assigns the glory of the craftsman to the work), as nearly all the diversity of motions are caused by a simple, magnetic and corporeal force, just as all the motions of a clock are caused by the simplest weight.” —Johannes Kepler (Snobelen 2012, p. 177)
- 3 “[T]he system of concepts that is unambiguously coordinated with the world of experience is reducible to a few basic principles, from which the whole system can be logically derived. With every important new advance the scientist finds his expectations exceeded, in that the fundamental laws, under the pressure of experience, become more and more simplified. With amazement he beholds the apparent chaos fitting into a sublime order that is to be attributed not to the dominion of his own mind but to the constitution of the world of experience.” —Albert Einstein (Cassirer [1936] 1956, p. 67)
- 4 Einstein, Podolsky, and Rosen 1935, p. 777.
- 5 Deutsch 1985, p. 4.
- 6 Hawking and Mlodinow 2010b, p. 70.
- 7 “[Nature’s] fundamental laws do not govern the world as it appears in our mental picture in a very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrelevancies. The formulation of these laws requires the mathematics of transformations.” —Paul Dirac ([1930] 1958, p. vii)
- 8 “Physics is an attempt conceptually to grasp reality as it is thought independently of its being observed. In this sense one speaks of ‘physical reality.’” —Albert Einstein ([1949] 1970, p. 81)
- 9 “But like the *Rain-bow* in a *Cloud* of mine. / Yet there’s a *Law* by which I *discompose* / The *Ashes*, and the *Fire* it self *disclose*, / But in his *Emerald* still *He* doth appear, / They are but *Grave-clothes* which he scatters here. / Who sees this *Fire* without his *Mask*. his *Eye* / Must needs be *swallow’d* by the *Light*, and *die*.” —Eugenius Philalethes [Thomas Vaughan] (Philalethes 1650, p. 54)
- 10 “The very word ‘discovery’ has something bluntly un-Classical in it. Classical man took good care not to take the cover, the material wrapping, off anything cosmic, but to do *just this* is the most characteristic impulse of a Faustian nature.” —Oswald Spengler ([1918] 1926a, Ch. VIII: Music and Plastic, II. Act and Portrait, §IV, p. 278)

that hidden mechanism,<sup>11</sup> which, for all of its intricacies, may be mysterious, but is never irregular nor inconsistent.<sup>12,13</sup>

[T]he heavenly bodies move with such regularity, orderliness, and symmetry that it is truly a marvel; and they continue always to act in this manner ceaselessly, following the established system, without increasing or reducing speed and continuing without respite [...]  
—Nicole Oresme<sup>14</sup>

*Confounding  
Factors*

Faustian culture comprehends the challenges of modeling and measurement exclusively in terms of the presence of ‘confounding factors’ which impede one’s clear view of objective fact.<sup>15,16,17</sup> Whenever a model does not accord with reality, it is supposed to be so because there exist these additional dynamical factors not yet accounted for—either intentionally or otherwise—just as the movements of a clock are surprising when there exists some corner of the internal mechanism that is as-yet unexplored.<sup>18,19,20</sup> My own perspective presents me with a particular, limited view of the universe, and all that I perceive is merely the appearance

11 “The specific tendency of all Western mechanics is toward an intellectual *conquest by measurement*, and it is therefore obliged to look for the essence of the phenomenon in a system of constant elements that are susceptible of full and inclusive appreciation by measurement [...]” —Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §I, p. 377)

12 “[N]or are we to recede from the analogy of Nature, which is wont to be simple, and always consonant to itself.” —Isaac Newton ([1687] 1934b, Rule III, Rules of Reasoning in Philosophy, Book III, p. 398)

13 “Subtle is the Lord, but malicious He is not.” —Albert Einstein (Pais [1982] 2005, p. vi)

14 Oresme [1377] 1968, p. 282

15 “The experiment is to be freed from all accidental circumstances that were a disturbing factor in the beginning; it is to be purged of possible experimental errors.” —Ernst Cassirer ([1936] 1956, p. 86)

16 The sentiment is that reality gets in the way of seeing reality. Cf. “No one can say that intelligibility is impeded by corporeal matter alone. For if that were the case, since matter is called corporeal only insofar as it is subject to a corporeal form, it would follow that matter has the characteristic of impeding intelligibility from the corporeal form. But this cannot be, because the corporeal form itself is actually intelligible, like other forms, insofar as it is abstracted from matter. Hence in no way is there composition of matter and form in the soul or in the intelligence in such wise that essence in them would be understood as it is in corporeal substances.” —Thomas Aquinas ([1252–1256] 1998, Ch. 4)

17 “And therefore what is sensuously felt, what is very significantly designated the plenum [...], is felt as a fact of the second order, as something questionable or specious, as a resistance that must be overcome by philosopher or physicist before the true content of Being can be discovered.” —Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §VII, p. 398)

18 “To study nature is to search into [His] workmanship: every new discovery opens to us a new part of his scheme.” —Colin Maclaurin ([1748] 1971, p. 3)

19 “If an explosion occurs when a ball is dropped, we say that some phenomenon must have occurred to make the cause proportional to the effect. On hunting for the phenomenon and not finding it, we say that it has merely not yet been found.” —Ludwig Wittgenstein ([1932–1935] 2001, p. 16)

20 “Every expansion of knowledge due to the steady increase of observational data and the refinements of measuring instruments meets with a corresponding characteristic simplification.” —Ernst Cassirer ([1936] 1956, p. 37)

of the objective reality,<sup>21</sup> in the same way that one's particular location in space and time prevents one from knowing the true motion of an object in absolute space.<sup>22</sup>

Every model of the world is understood to be a convenient<sup>23</sup> *approximation*<sup>24,25,26</sup> of the underlying reality, in that some more or less important details are left out of it,<sup>27</sup> and it is even called 'incorrect' therefore,<sup>28,29</sup> as though adding further detail to any picture of reality should bring it closer to *being* reality. So one might construct a perfectly reasonable model of a car without including its cup holders, and then add the cup holders as a marginal improvement later—and we are led to speak as though the difference between the model and the system itself ('reality') were contained in these very details.<sup>30</sup> We hold the entire world then to be ultimately comprehensible by such iterative analysis

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- 21 "It is one of the tacit, but none the less firm, presuppositions of nature-research that 'Nature' [...] is the same for every consciousness and for all times." —Oswald Spengler ([1918] 1926a, Ch. III: The Problem of World-History, I. Physiognomic and Systematic, §V, p. 103)
- 22 "But because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them. For from the positions and distances of things from any body considered as immovable, we define all places; and then with respect to such places, we estimate all motions, considering bodies as transferred from some of those places into others. And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be referred.¶ [...] It is possible, that in the remote regions of the fixed stars, or perhaps far beyond them, there may be some body absolutely at rest; but impossible to know, from the position of bodies to one another in our regions whether any of these do keep the same position to that remote body; it follows that absolute rest cannot be determined from the position of bodies in our regions." —Isaac Newton ([1687] 1934a, Scholium to the 'Definitions,' p. 8–9)
- 23 "[A]ll general physical concepts and laws [...] are obtained through idealization. In this way they take on that simplified form [...] which makes it possible to reconstruct and therefore understand any fact even of a complex nature by means of a synthetic combination of these concepts and laws." —Ernst Mach ([1905] 1976, p. 140)
- 24 "Physical laws rest on atomic statistics and are therefore only approximate [...]" —Erwin Schrödinger ([1944] 1992, p. 10)
- 25 "[I]n order to understand physical laws you must understand that they are all some kind of approximation." —Richard Feynman (R. P. Feynman, Leighton, and Sands [1963] 2010, Ch. 12–1)
- 26 "The key step is the selection, from the entire universe, of a subsystem to study. The key point is that this is always an approximation to a richer reality." —Lee Smolin (2013, p. 39)
- 27 "One after another, the characteristic features of time have proved to be approximations, mistakes determined by our perspective, just like the flatness of the Earth or the revolving of the sun. The growth of our knowledge has led to a slow disintegration of our notion of time. What we call 'time' is a complex collection of structures, of layers. Under increasing scrutiny, in ever greater depth, time has lost layers one after another, piece by piece [in the history of physics]." —Carlo Rovelli (2018, Prologue: Perhaps time is the greatest mystery, p. 4)
- 28 "Since all models are wrong the scientist cannot obtain a 'correct' one by excessive elaboration. On the contrary following William of Occam he should seek an economical description of natural phenomena." —George E. P. Box (1976, p. 792)
- 29 "We cannot remove a system from the universe, so in any experiment we can only decrease, but never eliminate, the outside influences on our system. In many cases, we can do this accurately enough to make the idealization of an isolated system a useful intellectual construct." —Lee Smolin (2013, p. 39)
- 30 "In the same way it is necessary to introduce new constants, such as the dielectric constant, when the behavior of gases in the presence of electric or magnetic fields is studied. And each such introduction of a new factor brings about a closer approximation to reality." —Ernst Cassirer ([1936] 1956, p. 86)

and measurement.<sup>31,32</sup> In the Faustian view, the world is merely *very* complex and difficult to comprehend, and it is that way because it comprises *so many* sub-systems.

*Mechanical and  
Non-Mechanical  
Models*

Classical mechanics—in Western culture the apotheosis not only of a physical theory, but of nature-knowledge itself<sup>33,34</sup>—provides the base framework<sup>35</sup> for constructing our pictures of the world, in which every physical ‘system’ behaves like a machine<sup>36</sup>, with every physical system comprising some number of dynamical bodies, each of which is itself a collection of smaller bodies obeying the same physical laws as the larger whole.<sup>37</sup> Indeed, it is a feature of every machine that each of its constituent parts is a machine in its own right: the engine in a car is a smaller machine than the car itself (if you place an engine on a hill, then it will roll down the hill just like a whole car), and star systems are decomposable in the same way—lunar dynamics are like planetary dynamics, are like galactic dynamics, etc.<sup>38</sup> (The theory of classical mechanics was developed with machines and planetary motion in mind, and Isaac Newton’s explicit goal in its canonical formulation was the description of the dynamics of these two particular physical domains according to a shared set of principles.)<sup>39,40</sup> Remove one planet from the Solar System,

31 “The assumption that we can increase our measurements beyond any limitation of precision is ultimately always based on the presupposition of infinite divisibility.” —Ernst Cassirer ([1936] 1956, p. 176)

32 “The world is constituted progressively and is finally—with respect to Nature as its abstractable component—constituted according to horizons in which something existent is constituted as actual in being-possibilities predesignated at any time [ . . . ]” —Edmund Husserl ([1934] 1981, p. 223)

33 “[T]he final aim of Natural Science is to discover the motions underlying all alteration, and the motive forces thereof; that is, to resolve itself into Mechanics.” —Hermann von Helmholtz (Spengler [1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §I, p. 377)

34 “For us there exists nothing but mechanical knowledge, no matter how miserable a substitute it is for true knowledge, and accordingly only one true form of scientific thought, that of mathematical physics.” —Emil du Bois-Reymond (Cassirer [1936] 1956, p. 6)

35 “But this does characterize the picture, the fact, namely, that it can be *completely* described by a *definite* net of definite fineness.” —Ludwig Wittgenstein ([1921] 1922, §6.342)

36 “The entire universe is a machine in which everything is made by figure and movement.” —René Descartes (Barbour [1989] 2001, p. 409)

37 “If this principle [of the reductive analysis of physical systems] were not valid, then we could never make any predictions about any experiment without knowing everything about the universe.” —Steven Weinberg (2004, p. 177)

38 The Rutherford–Bohr planetary model of the atom is a clear example of a misguided attempt to apply this paradigm at the quantum scale. (Bohr 1913; Rutherford 1911)

39 “Progress was much rather initiated by Newton’s at once applying the basic laws discovered by Galileo to the motion of what is most remote from us, namely celestial bodies; for it was precisely on this path that Newton found those tensions and completions of Galileo’s laws which in turn could be applied to more complicated terrestrial motions, so that he succeeded in working out a theory of the motion of bodies so perfect that to this day it has become the foundation not only of mechanics but of the whole of theoretical physics.” —Ludwig Boltzmann ([1900–1902] 1974d, p. 130)

40 “Modern physics studies, in the first line, the motion of ponderous bodies, i.e., the motion of bodies which surround us. Thus it is from the effort to explain the facts and the phenomena of common,

and all of the rest of the planets will continue along their paths more or less unchanged, as I can open up a clock, take out any part and replace it later without great consequence (but if I want to measure the weight of a mouse's liver, however, then I should first have to kill the mouse, or close to it<sup>41,42</sup>).

We say that these **Mechanical** models are the only true pictures of the world, the universe being nothing but a great machine,<sup>43</sup> and that these principles of Mechanics thereby represent reality itself;<sup>44</sup> non-Mechanical models (i.e., models that do not obey the same principles) are described as nothing more than expedient for use in the absence of the true Mechanical theory or sufficient observational data. One is inclined to say that it is merely *harder* to apply the principles of Mechanics in these cases, of course. Yet the principles of Mechanics, while very effective within the particular domain for which they were developed,<sup>45</sup> are totally useless at describing even such a mundane phenomenon as a

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every-day, experience—the act of falling, the act of throwing—that proceeds the trend of ideas which leads to the establishment of its fundamental laws. Yet it does not proceed therefrom exclusively, or even principally, or in a direct way. Modern physics does not originate from earth alone. It comes, just as well, from the skies. And it is in the skies that it finds its perfection and end.¶ This fact, the fact that modern physics has its 'prologue' and its 'epilogue' in the skies, or, to speak a more sober language, the fact that modern physics takes its origin from the study of astronomical problems and maintains this tie throughout its history, has a deep meaning, and carries important consequences. It expresses the replacement of the classic and medieval conception of the Cosmos—closed unity of a qualitatively determined and hierarchically well ordered whole in which different parts (heaven and earth) are subject to different laws—by that of the Universe, that is of an open and indefinitely extended entirety of Being, governed and united by the identity of its fundamental laws; it determines the merging of the *Physica coelestis* with *Physica terrestris*, which enables the latter to use and to apply to its problems the methods—the hypothetic-deductive mathematical treatment—developed by the former;" —Alexandre Koyré ([1968] 1992, pp. 1–2)

41 "In living nature nothing happens that is not in connection with a whole." —Johann Wolfgang von Goethe ([1792] 2010, p. 22)

42 "Thus, we should doubtless kill an animal if we tried to carry the investigation of its organs so far that we could tell the part played by the single atoms in vital functions. In every experiment on living organisms there must remain some uncertainty as regards the physical conditions to which they are subjected, and the idea suggests itself that the minimal freedom we must allow the organism will be just large enough to permit it, so to say, to hide its ultimate secrets from us." —Niels Bohr (1933)

43 "All physicists agree that the problem of physics consists in tracing the phenomena of nature back to the simple laws of mechanics." —Heinrich Hertz ([1894] 1899, p. xxi)

44 "If therefore we can liberate ourselves from an old and unfounded prejudice and from the lazy philosophy that tries to hide a sluggish lack of knowledge behind a pious face, then I hope to found a sure conviction on incontrovertible grounds: that the world recognizes a mechanical development out of the universal laws of nature as the origin of its constitution; [..]" —Immanuel Kant ([1755] 2012, p. 282)

45 "We are fortunate that the principles of Newtonian mechanics could be developed and verified to great accuracy by studying astronomical phenomena, where friction and turbulence do not complicate what we see." —E. T. Jaynes (2003, §10.8, p. 329)

leaf floating on a river.<sup>46,47,48</sup> (Where are the action and reaction? Where is the inertia? Where is the center of mass?) One imagines plenty of circumstances under which Mechanical principles are not applicable, as there are many objects that could not be described as falling at the same rate independent of their mass in accordance with the Equivalence Principle—fire, birds in flight, and flowing water all being simple examples.

A turbulent fluid,<sup>49</sup> indeed, comprises no distinct, enumerable ‘parts’ that themselves act like smaller versions of the same thing. Insofar as a fluid does have constituent elements, these elements are of a sort different from the greater whole (i.e., the molecules of a fluid are not themselves fluids—they do not, individually, take the shape of their container, for example).<sup>50</sup> So if someone asks me to explain the motion of a clock, I may answer by describing the motion of its parts; but if someone asks me to explain the motion of a fluid, I would not answer by describing the motions of its atoms: even when the dynamics of a small number of atoms may be described according to certain physical laws, it does not follow that a *very large* number of atoms may be described in the same way, despite the fact that there is no magic number which distinguishes ‘small’ and ‘large’ in any given context.<sup>51,52</sup>

46 “How far we are from actually solving all these equations, that is being able in all cases to obtain from them a genuinely clear picture of the processes in question, a simple glance at a foaming brook or at the water waves churned up by a large steamer will tell. How impotent analysis really is to read from the hydrodynamic equations the details of all these phenomena!” —Ludwig Boltzmann ([1900–1902] 1974d, p. 131)

47 “Unfortunately, there are really very few problems which can be solved exactly by analysis. In the case of the harmonic oscillator, for example, if the spring force is not proportional to the displacement, but is something more complicated, one must fall back on the numerical method.” —Richard Feynman (R. P. Feynman, Leighton, and Sands [1963] 2010, Ch. 10–1)

48 “Logically, the problem is one of inference (i.e., plausible conjecture) rather than deduction, since in almost all real problems of physics, biology, and economics our information is far too meager to permit any deductive proof that our predictions must be right. Indeed, it is often too meager to justify any definite predictions at all.” —E. T. Jaynes (1985, p. 254)

49 “I am an old man now, and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic.” —Horace Lamb (Goldstein 1969, p. 23)

50 “The atomist is mistaken only in that he assumes *mechanical* atoms, i.e., the finitude of *mechanical* divisibility.” —Friedrich Wilhelm Joseph von Schelling ([1799] 2004, p. 20)

51 Cf. the sorites paradox (Hyde and Raffman 2018).

52 Non-Mechanical systems include those that are qualitatively *too large*, *too small*, *too complex*, etc. *qua* machines. (If you imagine a machine with too many constituent elements, you are now talking about a statistical mechanical system; if you consider a too-small machine, you are dealing with quantum mechanics; if you take a system that is too complex to be described as a machine, you are now studying biology; and so forth.) Cf. “Indeed, the essential non-analyzability of atomic stability in mechanical terms presents a close analogy to the impossibility of a physical or chemical explanation of the peculiar functions characteristic of life.”¶ In tracing this analogy, however, we must remember that the problems present essentially different aspects in atomic physics and in biology. While in the former field we are primarily interested in the behaviour of matter in its simplest forms, the



This is not merely a practical limitation,<sup>53</sup> as it would be with a large and complex machine.<sup>54</sup> You could stare at a turbulent river for an eternity and never come close to being able to predict its behavior even minutes in advance, as it has no constituent parts to enumerate iteratively. That is, one wouldn't try to improve upon one's model of a turbulent fluid by adding a few more molecules here and there in order better to account for each of the discrepancies in any given description. The success of our Mechanical models at making predictions in the simplest cases of non-Mechanical systems (laminar flow, the two-body problem,<sup>55</sup> harmonic oscillators, etc.) is seen as an indication of the universality of Mechanical principles; yet, insofar as those cases are exceptional, they in fact demonstrate the general inapplicability of the Mechanical paradigm to the domains in question.<sup>56</sup>

So, the difference between *throwing a sandbag*, on the one hand, and *rolling a die*, on the other, seems to be merely a matter of degree—of the size of the moving body, the elasticity of its collisions with the floor and so forth. Yet the canonical models that we use to describe these two systems are different not only in what they depict but also in how they depict it. . . they have different grammars and they adhere to entirely different physical principles. The model of the sandbag treats the force with which the bag is thrown and its mass, friction, elasticity, etc.; it shows how the sandbag will roll, where it will land, and what side of the bag will be facing upwards when it comes to rest. The model of the die roll, however, tends to look more like, “I hold the die

*Determinism,  
Chaos, and  
Prediction*

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complexity of the material systems with which we are concerned in biology is of a fundamental nature, since even the most primitive organisms contain large numbers of atoms.” —Niels Bohr (1933)

- 53 “A closed hydrodynamical system of finite mass may ostensibly be treated mathematically as a finite collection of molecules—usually a very large finite collection—in which case the governing laws are expressible as a finite set of ordinary differential equations. These equations are generally highly intractable, and the set of molecules is usually approximated by a continuous distribution of mass.” —Edward Lorenz (1963, p. 130)
- 54 One cannot know the exact position and momentum of all of the atoms in a fluid, in precisely the same way that one cannot construct a perpetual motion machine. *Neither* is ‘physically realizable’ (Arora and Barak 2009, p. 26).
- 55 The  $n$ -body problem is provably unsolvable for  $n \geq 3$ , but we think of classical mechanics as describing the natural laws for all such systems, just as though  $n = 2$  were the norm and  $n \geq 3$  were instead the exception. (Cf. Gowers, Barrow-Green, and Leader (2008, pp. 495–496).)
- 56 “For essentially every physical theory we have (Navier-Stokes theory, general relativity, quantum field theory, etc.), we have very little detailed knowledge of the structure of generic solutions. Usually, we know exact solutions only under conditions of perfect or near-perfect symmetry or some other unrealistic assumption (two bodies, no external perturbative influences, [. . .]), and then argue that we can apply such solutions to real physical systems, because the approximation is adequate in appropriately controlled circumstances. That is to say: we have no real idea *at all*, in a representational sense, ‘what the world would be like if the theory were true or largely true of it’, for any physical theory.” —Erik Curiel (2019, p. 3)

in my hand and then throw it. The die bounces until eventually it comes to a stop, with one of its sides facing up at random.”

The model for the sandbag toss is called ‘deterministic,’ as we say that its future state is determined by its ‘initial conditions,’ i.e. the values for the various model parameters. This means that the same initial conditions will lead to the same result, where one means by ‘the same state’ here that any differences in their behaviors are *small* (whatever the standard for precision may be).<sup>57</sup> So if one were to throw a second sandbag that is a few milligrams heavier than the other, one would still expect it to land only some number of millimeters away. Really, there is no way of even describing the position of a sandbag so precisely.

By contrast, the die roll is an example of a ‘non-deterministic’ model: given the same initial state (‘I hold the die in my hand’), the result is not knowable in advance—it is unpredictable. But the probability of a die roll is commonly said to be nothing more than a feature of the manner in which we happen to understand a fundamentally deterministic underlying mechanism.<sup>58,59</sup> The idea is that if we were to throw two dice in ‘*exactly* the same way,’ then the result would be the same, and it’s only when the initial states of the two dice differ by a ‘small’ amount that the results should vary wildly. Predicting the outcome of a die roll using a deterministic model is supposed to be simply much *more* difficult than predicting the outcome of the sandbag toss, and we only use the model of a random die roll due to the difficulty of constructing a Mechanical model with all of the necessary parameters. But what standard of precision *could* there be for the specification of the initial state of the die roll?

The philosophical problem here is visible in the very definition of a ‘chaotic’ physical system, which is traditionally described as

<sup>57</sup> “‘Inexact’ is really a reproach, and ‘exact’ is praise.” —Ludwig Wittgenstein ([1953] 1958, §88)

<sup>58</sup> “Random events do not, therefore, exist in classical physics. Randomness is an appearance originating from the ignorance of initial conditions or from the inability to measure them or to compute explicitly the motion. These three kinds of limitation occur together when one throws dice, and this is why the result is said to be random, although it is not in principle.” —Roland Omnès (1999, p. 43)

<sup>59</sup> “[In the future, it will be the case that b]elief in ‘physical probabilities’ expressing a volition of the coin is recognized finally as an unfounded superstition. The existence of an underlying mechanical theory is proved beyond question; and the long success of the previous statistical theory is seen as due only to the lack of control over the initial conditions of the tossing. ¶ Because of recent spectacular advances in the technology of experimentation, with increasingly detailed control over the initial states of individual atoms, we think that the stage is going to be set, before very many more years have passed, for the same thing to happen in quantum theory; a century from now the true causes of microphenomena will be known to every schoolboy and, to paraphrase Seneca, they will be incredulous that such clear truths could have escaped us throughout the 20th (and into the 21st) century.” —E. T. Jaynes (2003, §10.8, pp. 329–30)

a “deterministic system that is sensitive to initial conditions”:<sup>60</sup> if a deterministic system is one where the same initial state (i.e., an insignificant difference in initial state) leads to the same result (i.e., an insignificant difference in the result), and a chaotic system is one where an arbitrarily small difference<sup>61</sup> in initial state (i.e., the same initial state) leads to a different result (i.e., a *large* difference in the result), then what could it mean for a chaotic system to be deterministic?

‘Chaotic’ models such as the three-body problem,<sup>62,63</sup> the Duffing oscillator,<sup>64</sup> and the double pendulum<sup>65</sup> in their very specifications make wildly different predictions for any possible level of precision.<sup>66</sup> The distinction that is drawn here between ‘similar’ and ‘exactly the same’ is a distinction without a difference, introduced only to unite the concepts ‘don’t know’ and ‘can’t know,’ since the unknown behavior, in a Mechanical picture of the world, is expected to be nothing but a matter of one’s own ignorance rather than a feature of the model in question. The unpredictability of the die roll and other non-deterministic models (those of fluid dynamics, for instance)<sup>67</sup> seems to be the result of the contingencies of our circumstance—a consequence of imprecision in observation and practical challenges in obser-

60 “Poincaré was also the first person to glimpse the possibility of **chaos**, in which a deterministic system exhibits aperiodic behavior that depends sensitively on the initial conditions, thereby rendering long-term prediction impossible.” —Steven Strogatz (2018, pp. 2–3)

61 “Suppose  $\mathbf{x}(t)$  is a point on the [Lorenz] attractor at time  $t$ , and consider a nearby point, say  $\mathbf{x}(t) + \delta(t)$ , where  $\delta$  is a tiny separation vector [...]. One finds that

$$\|\delta(t)\| \sim \|\delta_0\|e^{\lambda t}$$

where  $\lambda \approx 0.9$ . Hence *neighbouring trajectories separate exponentially fast.*” —Steven Strogatz (2018, §9.3)

62 Gowers, Barrow-Green, and Leader 2008, pp. 495–496.

63 The three-body problem is non-Mechanical because it cannot be divided into multiple, independent sub-problems like, say, the two-body problem (i.e., the separation between the two bodies and the motion of their center of mass).

64 Duffing 1918.

65 Shinbrot et al. 1992.

66 Indeed, there is a critical difference in how we describe deterministic systems that are and are not ‘sensitive to initial conditions’: we wouldn’t say that we have to know the *exact* parameters of the sandbag throw to predict its behavior. . . unless of course we wanted to predict its behavior *exactly*. That is, the die roll is not *more* difficult to observe in the same way a sandbag being thrown *faster* is. To put it another way, if I allow myself to observe the die roll with sufficient precision so as to be able to predict its behavior, then I must *also* allow the die roll to move too quickly to be measured with sufficient precision. (“Can God create a rock so heavy that He cannot lift it?”) In order to predict the outcome of a die roll, we should in fact have to make qualitative, rather than quantitative, changes to our model and in such a way that we deny the existence of the very difficulty that we are attempting to penetrate.

67 “Lack of periodicity is very common in natural systems, and is one of the distinguishing features of turbulent flow. [...]. It is sometimes possible to obtain particular solutions of these equations analytically, especially when the solutions are periodic or invariant with time [...]. Ordinarily, however, nonperiodic solutions cannot readily be determined except by numerical procedures.” —Edward Lorenz (1963, p. 130)

vation.<sup>68</sup> One says that the die roll is merely very complicated in its mechanism,<sup>69</sup> and until one comprehends it, its behavior merely *seems* random,<sup>70</sup> but that of course the result was in some sense determined beforehand.<sup>71,72,73</sup>

Imagine I present you with a sealed box and say, “This machine prints out random numbers, and I’ll bet you that it will print out an even number next.” How would you know that I had not rigged the device to output certain numbers in a particular order? Even if you see the machine print out a long string of digits in which no pattern is discernible, how could you know that inside the box there is no rotating drum etched with a code, as in a player piano, where the code determines in advance which number the machine will print out next? Were that the case, one *would* say that the box’s behavior merely ‘seems’ unpredictable, though it actually isn’t, because of the hidden internal mechanism. So when people gamble, they don’t employ such a machine to produce the winning numbers.<sup>74</sup> You can open up a machine, look inside it, and come to understand thereby absolutely everything that will ever happen with it; but can you examine the inside of a die (or of my throwing arm) with a microscope and discover a code that spells out which side the die will land on? The behavior

68 “In practice, only mathematical models have ideal symmetry and thus perfect degeneracy. Almost all physical systems are subject to interactions across vast ranges of scale from strong to weak. The ‘unpredictable’ directions of the order parameter are in real cases usually set by small asymmetries in the boundary conditions. For frost on a windowpane these may be slight impurities or dust. Such small effects are not sufficient to interfere with the reductionistically determined properties of order such as the strength of the ice matrix, and for the matrix as for the melted liquid state they would be un-noticeable. Only the degenerate properties can amplify such weak effects into large-scale macroscopic patterns.” —Smith and Morowitz (2016, p. 467)

69 “That which is wonderful in science is contrary to that which is wonderful in the art of the juggler. For the latter would wish to make us believe that we see a very simple causality, where, in reality, an exceedingly complex causality is in operation. Science, on the other hand, forces us to give up our belief in the simple causality exactly where everything looks so easily comprehensible and we are merely the victims of appearances. The simplest things are *very* ‘complicated’—we can never be sufficiently astonished at them!” —Friedrich Nietzsche ([1881] 1911, §6)

70 “‘How does thought manage to represent?’—the answer might be ‘Don’t you really know? You certainly see it when you think.’ For nothing is concealed.” —Ludwig Wittgenstein ([1974] 1980b, p. 104)

71 “What does determinism profess?¶ It professes that those parts of the universe already laid down absolutely appoint and decree what the other parts shall be. The future has no ambiguous possibilities bidden in its womb; the part we call the present is compatible with only one totality. Any other future complement than the one fixed from eternity is impossible. The whole is in each and every part, and welds it with the rest into an absolute unity, an iron block, in which there can be no equivocation or shadow of turning.” —William James ([1884] 1956, p. 150)

72 “At certain moments nature makes a choice.” —Paul Dirac (Langevin 1934, p. 33)

73 “Does this idea mean that nature is really free to choose the outcome of an experiment?” —Lee Smolin (2013, p. 148)

74 Machines may act as *pseudo*-random number generators only.

of the box cannot be predicted unless the box is opened. We ask, is there always a box to be opened?

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 PREDICTION AND TIME
 

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*Statistics belong, like chronology, to the domain of the organic, to fluctuating Life, to Destiny and Incident and not to the world of laws [. . .]. And if, now, suddenly the contents of that field are supposed to be understood and understandable only statistically and under the aspect of Probability—instead of under that of the a priori exactitude which the Baroque thinkers unanimously demanded—what does it mean? It means that the object of understanding is ourselves. Nature “known” in this wise is the Nature that we know by way of living experience, that we live in ourselves.*

—Oswald Spengler<sup>1</sup>

*Probability*

In the presence of philosophical confusion around particular concepts, one is often tempted simply to deny the very existence of those concepts<sup>2</sup> and to try to replace the corresponding terms with alternatives that are supposed to be somehow less slippery and ambiguous (like a mathematical symbol)<sup>3</sup>. This has been done with ‘probability,’<sup>4</sup> in the same way that the very concept of ‘solidity’ has been questioned as part of the development of the atomic theory:<sup>5</sup>

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<sup>1</sup> Spengler [1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §XIV, pp. 421–422

<sup>2</sup> E.g., “I, at any rate, am convinced that [God] is not playing at dice.” —Albert Einstein (M. Born, H. Born, and Einstein [1916–1955] 1971, Letter 52) (One might answer this with “God plays dice every time that *anyone* does.”)

<sup>3</sup> “Because language is misleading, as well as because it is diffuse and inexact when applied to logic (for which it was never intended), logical symbolism is absolutely necessary to any exact or thorough treatment of our subject.” —Bertrand Russell ([1919] 1920, p. 205)

<sup>4</sup> “Probability is the most important concept in modern science, especially as nobody has the slightest notion what it means.” —Bertrand Russell (E. T. Bell 1945, p. 587)

<sup>5</sup> Cf. “What *is* a chair? Well, a chair is a certain thing over there. . . certain?, [sic] how certain? The atoms are evaporating from it from time to time—not many atoms, but a few—dirt falls on it and gets dissolved in the paint; so to define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt, or which atoms are paint that belongs to the chair is impossible. So the mass of a chair can be defined only approximately. In the same way, to define the mass of a single object is impossible, because there are not any single, left-alone objects in the world—every

We have been told by popular scientists that the floor on which we stand is not solid, as it appears to common sense, as it has been discovered that the wood consists of particles filling space so thinly that it can almost be called empty. This is liable to perplex us, for in a way of course we know that the floor is solid, or that, if it isn't solid, this may be due to the wood being rotten but not to its being composed of electrons. To say, on this latter ground, that the floor is not solid is to misuse language. —Ludwig Wittgenstein<sup>6</sup>

So it is that many physicists still hold to the belief that even quantum mechanical systems are fundamentally 'deterministic'<sup>7</sup> at some 'lower level'<sup>8</sup>—that there exist 'hidden variables'<sup>9</sup> which govern quantum mechanical behavior and make it merely appear unpredictable (or that all of the randomness 'averages out' in such a way that it doesn't matter, or that they don't know).<sup>10</sup> Ironically, this conviction itself entails the reification of the mathematical representations of different quantum mechanical possibilities, the very existence of which then presents us with the philosophical problem of their 'reality.'<sup>11</sup> So the quantum mechanical wave function, which represents the mere probabilities of various eventualities,<sup>12</sup> is supposed to be a 'real wave,'<sup>13,14</sup> and a 'superposition of states' is supposed to be a state itself;<sup>15</sup>

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object is a mixture of a lot of things, so we can deal with it only as a series of approximations and idealizations." —Richard Feynman (R. P. Feynman, Leighton, and Sands [1963] 2010, Ch. 12–1)

6 Wittgenstein [1958] 1969, p. 45.

7 Brans 1988.

8 Donadi and Hossenfelder 2022.

9 't Hooft 2016.

10 Sivasundaram and Nielsen 2016.

11 "Dynamic and statistical laws were not regarded as two complementary methods and directions, as two different modes of description; they were instead opposed as the 'determined' and the 'undetermined.' Thus the new problems that were introduced by the second law and by the development of quantum theory were discussed under the heading of 'indeterminism,' a title which gives rise to the most dangerous equivocations. It seemed to open the doors to a liberum arbitrium indifferentiae, a state of freedom which was hardly distinguishable from caprice." —Ernst Cassirer ([1936] 1956, p. 89)

12 Bohm 1952, p. 167.

13 "[D]oes a  $\psi$ -function of the quantum theory represent a real factual situation [...]?" —Albert Einstein ([1949] 1970, p. 83)

14 "No one can understand [the quantum theory] until he is willing to think of  $\psi$  as a real objective field rather than just a 'probability amplitude'. Even though it propagates not in 3-space but in  $3N$ -space." —J. S. Bell (1987, p. 128)

15 "At the heart of much debate concerning quantum theory lies the quantum state. Does the wave function correspond directly to some kind of physical wave? If so, it is an odd kind of wave, since it is defined on an abstract configuration space, rather than the three-dimensional space we live in. Nonetheless, quantum interference, as exhibited in the famous two-slit experiment, appears most readily understood by the idea that it is a real wave that is interfering. Many physicists and chemists

Schrödinger's cat is considered to be both dead *and* alive before its observation, instead of simply dead *or* alive.<sup>16</sup> The paradox arises when we then ask what this superposition is, even as we invented the very concept of 'superposition' in order to be able to describe quantum mechanical phenomena while avoiding the direct treatment of probability (and even as we use the existence of that invented concept as evidence of the absolute truth of the quantum mechanical theory<sup>17</sup>).

The same confusion exists outside of academic physics as well, where we impart intention and mystery to other quintessentially probabilistic phenomena. When you pick a card from a deck at random—that is, when you have no reason to pick one instead of another—it is, by definition, a matter of chance which one you choose. Still, it is hard to accept that there really is *no* reason why you should have picked a particular card. We see this in the practice of tarot reading, where the choosing of different cards from a shuffled deck may represent various prognostications about one's life, say. It is believed that the card one chooses is predetermined by some unknown (but not unknowable) mechanism. Now, ask an enlightened individual about tarot, and they will dismiss the whole idea as nothing but superstitious nonsense; but ask that same person whether or not it is *really* a matter of chance which side a die will land on when it is thrown, and you will undoubtedly hear some serious equivocation in response.

*Cause and Effect*

We say that the world is deterministic and also that time is directed;<sup>18</sup> but a model whose future state is perfectly predictable (i.e., knowable) is one for which there is no distinction between the past and the future.<sup>19</sup> Mechanical systems, in particular, are deterministic, predictable, and 'symmetric under time

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concerned with pragmatic applications of quantum theory successfully treat the quantum state in this way." —Pusey, Barrett, and Rudolph (2012)

16 "It is one of the most deep rooted mistakes of philosophy to see possibility as a shadow of reality." —Ludwig Wittgenstein ([1974] 1980b, p. 283)

17 "[E]very physical observation is so constituted that it *proves the basis of a certain number of imaged presuppositions*; and the effect of its successful issue is to make these presuppositions more convincing than ever." —Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §I, p. 378)

18 "'Time has only one direction' must be a piece of nonsense.¶ Having only one direction is a logical property of time." —Ludwig Wittgenstein ([1961] 1979, §12.10.16)

19 "[A]n intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes." —Pierre-Simon Laplace ([1825] 1902, p. 4)



reversal.<sup>20,21</sup> Indeed, the Laws of Mechanics give no indication as to the temporal order of events—they do not specify an ‘arrow of time.’<sup>22</sup> By contrast, a (probabilistic) die roll, and chaotic models generally, specify the direction of time insofar as the probability in the model draws a distinction between the state of the model in the future and the state of the model in the past (‘The die bounces until *eventually* it comes to a stop. . .’).

In Mechanics, there is no distinction between a weight pushing down on a spring and the spring pushing up on the weight: ‘cause and effect’ are eliminated with Newton’s Third Law, which establishes a perfect symmetry between action and reaction:<sup>23</sup> forces are no longer *causes* of motion, but instead merely (reversible) changes in motion.<sup>24,25</sup> Indeed, if you take an ice skater gliding on a flat rink, then a change in the motion of that skater, say when someone gives them a push, may be understood either (1) as a collision between the skater and the person pushing them, or (2) as a consequence of that push. The former is the picture from Mechanics.<sup>26</sup> So every point on the face of a clock represents (infinitely many) times in both the past and the future;<sup>27</sup> a

20 “The fundamental equations of mechanics do not in the least change their form if we merely change the algebraic sign of the time variable. All purely mechanical processes can therefore occur equally well in the sense of increasing and decreasing time. But we notice even in ordinary life that future and past do not correspond at all so perfectly as the directions right and left, that on the contrary they are clearly distinguishable.” —Ludwig Boltzmann ([1904] 1974b, p. 170)

21 “The second law of thermodynamics is the only ‘fundamental’ law (including in quantum physics) that breaks time-reversal invariance;” —Carlo Rovelli (2022, p. 1)

22 Eddington 1929, Ch. IV.

23 Cf. “When we analyze the basic phenomenon on which the mechanical explanation of nature is built, we find that precisely this phenomenon contains no shadow of a real explanation. [. . .]. If the two billiard balls at the instant of collision were to exchange their colors instead of altering their direction of motion, the one would be just as much and as little understandable as the other.” —Ernst Cassirer ([1936] 1956, p. 16)

24 “It may be said generally that since Galileo, classical physics has renounced the task of answering any question ‘why?’ and that it is only by virtue of this renunciation that it has been able to solve the specific task which it has undertaken. It no longer seeks to know why events occur but only that they occur and how they occur, and it is satisfied with establishing definite laws for both the nomological and the ontological structure of the world.” —Ernst Cassirer ([1936] 1956, p. 106)

25 “Before Galileo, the theory of terrestrial motions had been dominated by the concept of cause. The important thing was to find a qualitative explanation (in terms of essential nature etc.) of why any particular body moved in the way it did. Galileo by no means threw off this way of thinking entirely. Instead, he augmented it by an approach that, at least up to the present day, has proved to be far more fruitful. He stopped looking for *causes* of motion and instead, like the early astronomers, sought merely to *describe* actually observed motions. He no longer asked: *why* does the stone fall, but *how* does the stone fall? In the *Discorsi*, he comments that innumerable books had been written explaining why bodies fall towards the ground with an accelerated motion but adds drily that ‘to just what extent this acceleration occurs has not yet been announced’ ” —Julian B. Barbour ([1989] 2001, p. 357)

26 “In a system of bodies which conforms to the fundamental law there is neither any new motion nor any cause of new motion, but only the continuance of the previous motion in a given simple manner.” —Heinrich Hertz ([1894] 1899, p. 147)

27 I can draw a picture of the face of a clock to indicate at what time an event happened [in the past], or to indicate a time at which an event will happen [in the future], and there might not be any difference between those two drawings themselves.

pendulum swings both ways. And yet, a river will only flow in one direction ('downstream,' due to gravity).

A flowing river has a clear direction in time,<sup>28</sup> but could you measure quantities of time with one? How would you define an hour? You could place some markers an equal distance apart along the river's banks and throw a leaf into the water upstream of the first; then you could say that one hour passes whenever a leaf moves from one marker to the next. But how would you know that the river was moving at the same speed from marker to marker?<sup>29</sup> The river could be shallower at some point, with the water flowing faster there. Equivalently, how many markers would you have to put in place? You would need a large number in order to compensate for local turbulence; but the greater the number (the greater the distance), the more systemic bias there would be (e.g., due to variations in geography). A transverse slice of a river is not a river, and in such a slice there is precisely no upstream nor downstream; there is no past and no future. The motion of the water in any particular region of the river is turbulent, chaotic, and unpredictable.

By contrast, time as measured with a clock is readily quantified: if the distance between any pair of teeth on a gear of a clockwork mechanism is too wide or narrow, that introduces no bias in the operation of the mechanism. Indeed, one may measure time easily by the motions of the heavens: a 'day' is the period between two sunrises, a 'year' between two equinoxes, etc. (cf. a sundial). The planets repeat their motions indefinitely, as a pendulum swings frictionlessly, and as an hourglass may be turned over and over forever. In other words, a clock with a heavy pendulum, rigid gears, and no friction will never slow down: one may

28 "It is clear that this question most easily arises if we are preoccupied with cases in which there are things flowing by us,—as logs of wood float down a river. In such a case we can say the logs which have passed us are all down towards the left and the logs which will pass us are all up towards the right. (I might halt the motion of a pendulum clock simply by fixing the pendulum in place—what could a 'stopped river' refer to, by analogy? It would no longer be a river, but a lake, which is in fact divisible into parts.) We then use this situation as a simile for all happening in time and even embody the simile in our language, as when we say that 'the present event passes by' (a log passes by), 'the future event is to come' (a log is to come). We talk about the flow of events; but also about the flow of time—the river on which the logs travel." —Ludwig Wittgenstein ([1958] 1969, p. 107)

29 Cf. "If we look at a river in which numbered logs are floating, we can describe events on land with reference to these, e.g., 'When the 105th log passed, I ate dinner'. Suppose the log makes a bang on passing me. We can say these bangs are separated by equal, or unequal, intervals. We could also say one set of bangs was twice as fast as another set. But the equality or inequality of intervals so measured is entirely different from that measured by a clock." —Ludwig Wittgenstein ([1932–1935] 2001, §13)

always imagine a better clock, such that its repetition is all the more perfect.

We see the difference between the time-reversal symmetry of Mechanical systems and the temporal directedness of non-Mechanical systems thus: Consider a small number of particles bouncing around elastically in a container. The motions of these particles may be modeled according to Newton's Laws so that the entire behavior of the system both forward and backward in time may be fully comprehended. Now take the same system, but with uncountably many such particles, such that it may be treated as a gas<sup>30</sup> and with the science of statistical mechanics. The motions of the individual particles are then no longer part of that model, and the behavior of the aggregate adheres to different physical principles entirely—among others, the Second Law of Thermodynamics: we know that the entropy of the whole will tend toward a maximum,<sup>31</sup> whatever the microbehavior may be, and this manifests as a temporal asymmetry in the model (which is non-Mechanical<sup>32</sup>). The qualitative complexity of the model—represented by the scale of the number of particles in the box—determines whether the model is Mechanical or not, insofar as Mechanical models may be analyzed reductively as some number of constituent elements and *thereby* their behavior predicted.<sup>33</sup>

*Time in  
Mechanics*

Analogously, in the domain of living systems, Natural Selection proceeds by infinitesimal, incremental, and individually probabilistic steps (gene mutations, the reproductive success of individuals, etc.)—steps that, in large aggregates, provide for the directed process of evolution.<sup>34</sup> So, Natural Selection cannot explain any particular event in the history of a biological sys-

<sup>30</sup> Clausius 1857, p. 113.

<sup>31</sup> "Every process occurring in nature proceeds in the sense in which the sum of the entropies of all bodies taking part in the process is increased. In the limit, i.e., for reversible processes, the sum of the entropies remains unchanged." —Max Planck ([1897] 1903, §133)

<sup>32</sup> "[T]he Second Law by introducing irreversibility has for the first time brought into the mechanical-logical domain a tendency belonging to immediate life and thus in fundamental contradiction with the very essence of that domain." —Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §XIV, p. 421)

<sup>33</sup> "You can't *build* clouds. And that is why the future you *dream* of never comes true." —Ludwig Wittgenstein ([1977] 1980a, 41e)

<sup>34</sup> "Darwinian dynamics is a special case within the more general class of order-forming, non-equilibrium stochastic processes." —Smith and Morowitz (2016, p. 571)

tem,<sup>35,36,37</sup> just as the Second Law cannot explain the transition to any particular microstate of a thermodynamic system. The various stages of evolution that have led to any particular biological feature were not themselves directed toward ‘survival of the fittest,’<sup>38</sup> any more than the motions of individual particles in the gas-in-a-box<sup>39</sup> are directed toward the maximization of the entropy of the system as a whole,<sup>40</sup> which is in fact the cause of those events. Like statistical mechanical systems, living systems are qualitatively, not quantitatively, more complex than Mechanical ones.<sup>41,42</sup> (There are millions of bacteria on every square inch of your skin<sup>43</sup> and quadrillions of ants alive on the surface of the Earth,<sup>44</sup> as there are quintillions of molecules in every cubic centimeter of air at standard temperature and pressure.)

Non-Mechanical systems—such as thermodynamic and biological<sup>45</sup> ones—are in general irreducible: I cannot say precisely what will happen, and insofar as I don’t know what will happen, I do know that I will *learn* it, which constitutes the very temporal asymmetry itself.<sup>46</sup> So, one knows that a lottery machine will eventually eject *some* ball from its chamber—it is *which* ball it will

35 The very term ‘natural history’ shows the affinity between the study of living systems and that of history proper, which is also the study of Life. (“These cultures, sublimated life-essences, grow with the same superb aimlessness as the flowers of the field.” —Oswald Spengler ([1918] 1926a, Ch. I: Introduction, §7, p. 21))

36 “I have always thought that Darwin was wrong: his theory doesn’t account for all this variety of species. It hasn’t the necessary multiplicity. Nowadays some people are fond of saying that at last evolution has produced a species that is able to understand the whole process which gave it birth. Now that you can’t say.” —Ludwig Wittgenstein ([1984] 1996, p. 160)

37 Cf. “In calling the structure of the chromosome fibres a code-script we mean that the all-penetrating mind, once conceived by Laplace, to which every causal connection lay immediately open, could tell from their structure whether the egg would develop, under suitable conditions, into a black cock or into a speckled hen, into a fly or a maize plant, a rhododendron, a beetle, a mouse or a woman.” —Erwin Schrödinger ([1944] 1992, p. 21)

38 Cf. “the blind watchmaker” —Richard Dawkins ([1986] 1996, p. 9)

39 Gibbs [1875–1878] 1906, pp. 165–68.

40 “How can the irreversibility of macroprocesses be reconciled with the reversibility of microprocesses? It is this paradox which the physicist has to resolve when he wishes to account for the direction of thermodynamical processes and for the direction of time.” —Hans Reichenbach (1956, p. 109)

41 “And so, too, a corpse seems to us quite inaccessible to pain.—Our attitude to what is alive and to what is dead, is not the same. All our reactions are different.—If anyone says: ‘That cannot simply come from the fact that a living thing moves about in such-and-such a way and a dead one not’, then I want to intimate to him that this is a case of the transition ‘from quantity to quality’.” —Ludwig Wittgenstein ([1953] 1958, §284)

42 “Biology is hard because there’s so *much* of it.” —Randall Munroe (2020)

43 Gao et al. 2010.

44 Schultheiss et al. 2022.

45 “[N]ature, considered as mere mechanism, could have produced its forms in a thousand other ways without stumbling upon the unity [of organization] which is in accordance with such a principle [of purpose]. It is not in the concept of [mechanical] nature but quite apart from it that we can hope to find the least ground a priori for this.” —Immanuel Kant ([1790] 1914, p. 260)

46 “Both the sources of blurring—quantum indeterminacy, and the fact that physical systems are composed of zillions of molecules—are at the heart of time. Temporality is profoundly linked to blurring. The blurring is due to the fact that we are ignorant of the microscopic details of the world.

be that is a matter of chance. Whenever I roll a die, I can say definitely that it will eventually come to rest with *some* side facing up. While any particular current of water may twist and turn and even move upstream a short ways, the river as a whole—every river—flows ceaselessly down to the sea. (We can talk about a thermodynamical model of a clock wherein the clock will eventually run down or even decay; but one cannot predict when or how, and we would not be inclined to call such a prognostication a description of ‘how the clock will behave,’ which is what constitutes the scope of the original Mechanical picture.) We know the end, but not how that end will happen to be reached,<sup>47</sup> as with the flow of a turbulent river (*pace* Nietzsche<sup>48</sup>), but not the movements of a machine, for which one is always able to reduce the dynamics of the whole to those of the parts.<sup>49</sup>

The temporal directedness of natural processes, then, cannot be represented by our Mechanical models, where time is quantifiable, where there is no causality, and where all processes are perfectly reversible. The time that we measure with a clock—the time that we find in our theory of classical mechanics—is not the same time that exists, for example, in our models of thermodynamic, chaotic, and quantum mechanical phenomena. The decomposability and analyzability of Mechanical models allow for and require a *spatial* conception of time, a conception wherein ‘spans of time’ may be divided up and measured by, say, the ticks of a second hand that are laid end to end against an event, like rigid rods laid against a body in the measuring of its length.<sup>50</sup> And Mechanical time is, like space, undirected: with the dis-

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The time of physics is, ultimately, the expression of our ignorance of the world. Time is ignorance.” —Carlo Rovelli (2018, p. 140)

47 “The cause is hidden; but the [...] power of the fountain is well known.” —Ovid ([ca. 8] 1971, p. 199)

48 “At the sight of a waterfall we think we see in the countless curvings, twistings and breakings of the waves capriciousness and freedom of the will; but everything here is necessary, every motion mathematically calculable. So it is too in the case of human actions; if one were all-knowing, one would be able to calculate every individual action, like-wise every advance in knowledge, every error, every piece of wickedness. The actor himself, to be sure, is fixed in the illusion of free will; if for one moment the wheel of the world were to stand still, and there were an all-knowing, calculating intelligence there to make use of this pause, it could narrate the future of every creature to the remotest ages and describe every track along which this wheel had yet to roll. The actor’s deception regarding himself, the assumption of free-will, is itself part of the mechanism it would have to compute.” —Friedrich Nietzsche ([1894] 1996, p. 57)

49 “[I]t is absurd [...] to hope that another *Newton* will arise in the future, who shall make comprehensible by us the production of a blade of grass according to natural laws which no design has ordered.” —Immanuel Kant ([1790] 1914, pp. 312–13)

50 “The key point, strongly emphasized by Galileo, is that two well attested physical phenomena, the law of free fall and the law of inertia, make it possible to convert a purely geometrical ruler into a ‘ruler of speeds.’” —Julian Barbour ([1989] 2001, p. 516)

covery of the hidden details of a Mechanical model, the future may be predicted perfectly and all that has ever happened in the past may be seen clearly simply by ‘running the equations backwards.’<sup>51</sup> Newtonian time is absolute, universal, and objectively knowable as a fundamental feature of the very fabric of our reality,<sup>52</sup> just like Newtonian space.<sup>53</sup> In Relativity theory, of course, time is just another dimension of the space-time manifold, albeit one with an imaginary unit,<sup>54,55,56</sup> and causality is represented by nothing more than the relative position of events in that four-dimensional continuum.<sup>57</sup>

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- 51 “[W]e showed that many stars would eventually collapse and produce closed trapped surfaces. If one goes to a larger scale, one can view the expansion of the universe as the time reverse of a collapse. Thus one might expect that the conditions of theorem 2 would be satisfied in the reverse direction of time on a cosmological scale, providing that the universe is in some sense sufficiently symmetrical, and contains a sufficient amount of matter to give rise to closed trapped surfaces.” —Hawking and Ellis (1973, p. 348)
- 52 “Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration [...]” —Isaac Newton ([1687] 1934a, Scholium to the ‘Definitions,’ p. 6)
- 53 “Absolute, true, and mathematical space remains similar and immovable without relation to anything external.” —Isaac Newton ([1687] 1934a, Scholium to the ‘Definitions,’ p. 6)
- 54 “Without [the idea of Minkowski space-time] the general theory of relativity [...] would perhaps have got no farther than its long clothes.” —Albert Einstein ([1917] 1920, p. 57)
- 55 “One might think this means that imaginary numbers are just a mathematical game having nothing to do with the real world. From the viewpoint of positivist philosophy, however, one cannot determine what is real. All one can do is find which mathematical models describe the universe we live in. It turns out that a mathematical model involving imaginary time predicts not only effects we have already observed but also effects we have not been able to measure yet nevertheless believe in for other reasons. So what is real and what is imaginary? Is the distinction just in our minds?” —Stephen Hawking (2001, p. 59)
- 56 “All that has been said about time in ‘scientific’ philosophy, psychology and physics [...] touches, not at any point the secret itself, but only a spatially-formed representative phantom. The livingness and directedness and fated course of real Time is replaced by a figure which, be it never so intimately absorbed, is only a *line*, measurable, divisible, reversible, and not a portrait of that which is incapable of being portrayed; by a ‘time’ that can be mathematically expressed in such forms as  $\sqrt{t}$ ,  $t^2$ ,  $-t$ , from which the assumption of a time of zero magnitude or of negative times is, to say the least, *not* excluded.” —Oswald Spengler ([1918] 1926a, Ch. IV: The Problem of World-History, II. The Idea of Destiny and the Principle of Causality, §II, p. 124)
- 57 “Associated with each event,  $p$ , in spacetime is a light cone [...]. We assign the label ‘future’ to half of the cone and the label ‘past’ to the other half. The events lying in the interior of the future light cone represent events which can be reached by a material particle starting at  $p$ ; these comprise the ‘chronological future’ of  $p$ . The chronological future of  $p$  together with the events lying on the cone itself comprise the ‘causal future’ of  $p$ , which physically represents events which, in principle, can be influenced by a signal emitted from  $p$ .” —Robert M. Wald (1984, p. 188)

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## MOTION AND RELATIVITY

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*Every physics [...] must break down over the motion-problem, in which the living person of the knower methodically intrudes into the inorganic form-world of the known.*  
—Oswald Spengler<sup>1</sup>

When I say that an object is moving, I could mean that it is moving translationally, like a clock on a conveyor belt (say, as measured with a laser rangefinder), or I could mean that it is moving ‘internally,’ like a clock with a swinging pendulum or a clock spinning in place (as measured in terms of the change of the shape, internal configuration, or orientation of the clock—for example, “The little hand is now over the numeral 7.”), even as the clock itself does not change in its position at all.<sup>2,3</sup> One is able to describe each of these motions either as internal or as translational: so, the internal motion of the ticking clock is also the translation of the various parts of the clock—where each of the parts is itself moving as a whole—and a change in the distance between the clock on the conveyor belt and some other body (e.g., the conveyor belt itself) may be described as the internal motion of a composite body (the assembly line) of which the first and the second bodies are constituent parts.<sup>4</sup>

*Translational  
vs. Internal  
Motion*

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<sup>1</sup> Spengler [1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §XIII, p. 418

<sup>2</sup> Cf. “Hence the statement that the motion of a simple body is simple holds true in particular for circular motion, as long as the simple body abides in its natural place and with its whole. For when it is in place, it has none but circular motion, which remains wholly within itself like a body at rest. Linear motion, however affects things which leave their natural place or are thrust out of it or quit it in any manner whatsoever.” —Nicolaus Copernicus ([1543] 1978, Book 1, Ch. 8)

<sup>3</sup> Cf. “Moving force is of two kinds: either the locomotion of a body (*vis locomotiva*) which forces another to leave its place, or internal motion.” —Immanuel Kant ([1884] 1993, Fascicle VIII, sheet I, p. 1, later deleted by Kant)

<sup>4</sup> “Perhaps one might say that the difficulties would not arise if I and if we were able to fly and have two earths as basis-bodies, being able to arrive at the one from the other by flight. Precisely in this way the one body would be the basis for the other. But what do two earths mean? Two pieces of one earth



If I want to measure the speed at which the clock is moving as a whole, I might point a radar gun at the clock myself, or I might affix the radar gun to another object and read the results from a distance. In the latter case, one says, I'm measuring the speed of the clock *relative* to the second object, and we call that object the 'frame of reference' for that measurement. What is the frame of reference for a measurement of internal motion, however? When I observe the hands of a clock rotating around the center of the dial, does it matter where I make that observation from? We might say that one hand of the clock is moving toward or away from some point on the clock's face... indeed, I might attach lasers to the frame of the clock and point them at the hands to determine the time; but that is now a description of translation—of the motion of each part as a whole—and that is not how one normally reads a clock.<sup>5</sup>

With Mechanics, where all bodies are divisible and the dynamics of the whole are the same as the dynamics of its constituent parts, *there are no descriptions of internal motion*<sup>6,7,8,9</sup>—only translation is ever described, and even rotation is understood always as the *cyclical* (e.g., circular) motion of the parts of the whole, that is, as translation where the direction of motion is constantly

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with a humanity. Together, they would become one basis and, at the same time, each would be a body for the other. Surrounding them would be a common space in which each, as body, possibly would have a movable place, but motion would always be relative to the other body and nonrelative to the synthetic basis of their being together. The places of all bodies would have this relativity. However, one would always still have to ask, motion and rest with respect to which of the two basis bodies?" —Edmund Husserl ([1934] 1981, p. 227)

- 5 "Think of the recognition of *facial expressions*. Or of the description of facial expressions—which does not consist in giving the measurements of the face!" —Ludwig Wittgenstein ([1953] 1958, §285)
- 6 "[S]uppose a vortex together with all the Planets, or a ship along with everything within it floating in the sea, or a man walking in a ship together with the things he carries with him, or the wheel of a clock together with its constituent metallic particles. For unless you say that the motion of the whole aggregate cannot be considered as proper motion and as belonging to the parts according to the truth of things, it will have to be admitted that all these motions of the wheels of the clock, of the man, of the ship, and of the vortex are truly and philosophically speaking in the particles of the wheels." —Isaac Newton ([1684] 1962, pp. 126–127)
- 7 "Mechanics is the theory of the motion of natural bodies, that is change of place (relative change of position) that is not connected with any change of their other properties. According to this definition mechanics must also explore under what conditions a body does not change its place; that is, is at rest." —Ludwig Boltzmann ([1897] 1974a, p. 224)
- 8 "In modern science, as well we know, motion is considered as purely geometrical translation from one point to another. Motion, therefore, in no way affects the body which is endowed with it; to be in motion or to be at rest does not make any difference to, or produce a change in, the body whether in motion or at rest. The body, as such, is utterly indifferent to both." —Alexandre Koyré ([1968] 1992, p. 4)
- 9 "However, the reasoning used primarily by Newton [to justify his Third Law of Motion] applies to point particles without structure and is not concerned with the motion of material bodies composed with a large number of particles, in or out of thermal equilibrium." —Mario J. Pinheiro (2011, p. 1)



changing.<sup>10,11,12,13</sup> All motion is specified in terms of frames of reference.<sup>14</sup> So, instead of saying, for instance, “the clock is ten meters away,” we say “the clock is ten meters away from the lamp.” What we are describing is the space between the two objects<sup>15,16</sup> rather than any object itself.<sup>17</sup> Indeed, Mechanically, objects are modeled as simple bodies located at particular points in space: when I measure the distance between a clock and another body, then the result of that measurement depends on which part of the clock the laser on the second body is pointed at. For there to be only one answer, I should have to ignore any differences in the results, calling them insignificant;<sup>18</sup> in order to talk about the motion of a body toward or away from me, I should have to model it as internally at rest.<sup>19,20</sup>

- <sup>10</sup> “Just as a curve can be considered as consisting of an infinity of right lines, even if in truth it does not consist of them but because this hypothesis is useful in geometry, in the same way circular motion can be regarded as traced and arising from an infinity of rectilinear directions, which supposition is useful in the mechanical philosophy.” —George Berkeley ([1721] 1992, §61)
- <sup>11</sup> “Circular motion is relative motion along parallel lines with a continually changing direction and a distance remaining constant owing to a bond. ¶ Circular motion in one body is motion with respect to the parts, with the distance remaining constant owing to a bond. [Author’s Translation]” —Christiaan Huygens ([1629–1256] 1888–1950, p. 507)
- <sup>12</sup> “It seems that Newton regarded centripetal force as of greater importance than all the other forces; it occupied his mind more than anything else in mechanics and it was his real point of departure for all considerations of this chapter of definitions. Indeed, his remarks on this definition begin with the words: ‘Of this sort is gravity.’ ” —Max Jammer ([1957] 1962, p. 122)
- <sup>13</sup> “Just as it must be possible to write down any arbitrary number by means of the system of numbers, so it must be possible to write down any arbitrary proposition of physics by means of the system of mechanics.” —Ludwig Wittgenstein ([1961] 1979, §6.12.14)
- <sup>14</sup> “Now I begin to see that I lack something in the expression of motion and rest. I should never say, a body is at rest, without adding with regard to what it is at rest, and never say that it moves without at the same time naming the objects with regard to which it changes its relation.” —Immanuel Kant ([Jammer 1954] 1993, p. 132)
- <sup>15</sup> “The chief general propositions in mechanics [...] have all been developed from the assumption of Newton’s attributes of constant, and therefore conservative, forces of attraction between material points, and of the existence of fixed connections between them.” —Hermann von Helmholtz ([1894] 1899, p. xix)
- <sup>16</sup> “Mechanist physics had conceived all bodies as aggregates of material points acting directly on each other at a distance.” —Ludwig Boltzmann (1974e, p. 143)
- <sup>17</sup> “Consequently, we are unable to ascribe motion to a determined body considered in itself. A body is only in motion in its relation to something else—some other body—which we assume to be at rest. We can, therefore, ascribe it to the one or to the other of the two bodies, *ad libitum*. All motion is relative.” —Alexandre Koyré ([1968] 1992, p. 4)
- <sup>18</sup> “If one wishes to determine the distance of the moon to the earth, one wishes to know the distance of their places, and to this end one does not measure from any point of the surface, or of the inferior of the earth, to any point of the moon at pleasure, but takes the shortest line from the central point of the one to the central point of the other, and therefore, in each of these bodies there is only one point that constitutes its place.” —Immanuel Kant ([1786] 1909, p. 152)
- <sup>19</sup> “595. 2. Those masses whose position still remains unknown when the coordinates accessible to observation have been completely specified are called concealed masses, their motions concealed motions, and their coordinates concealed coordinates. In contradistinction to these the remaining masses are called visible masses, their motions visible motions, and their coordinates visible coordinates. ¶ 599. 6. Continually recurrent motions, and therefore cyclical motions, are frequently concealed motions; for these, when existing alone, cause no change in the mass-distribution, nor therefore in the appearance of things.” —Heinrich Hertz ([1894] 1899, pp. 223–224)
- <sup>20</sup> “Hertz said that wherever something did not obey his laws there must be invisible masses to account for it. This statement is not right or wrong, but may be practical or impractical. Hypotheses such as

Distinguishing between translational and internal motion in this manner helps to elucidate the nature of *forces*<sup>21</sup> and its relation to motion generally.<sup>22,23</sup> We ask, what does it mean to say “a body feels a force?” On the one hand, Newton says that the ‘force’ felt by a body is proportional to the change in motion of that body, for instance a point particle moving in an electric field;<sup>24</sup> and on the other hand, a vertically oriented spring compressed under a weight does not move at all, and yet we should still say that the spring is feeling a force. (We say that, in the latter case, the downward force is counteracted by an upward force from the table, ‘normal to its surface,’ keeping the spring from moving.) But are the forces in these two cases the same kind of thing? How are they each measured?<sup>25</sup>

Consider a train accelerating from a stop, with a passenger sitting in a seat facing forward and a steward on roller skates standing in the aisle (Figure 1). As the train moves forward, the steward rolls backward in the train and the passenger is compressed by the chair. How could the steward know that the train is moving? He could see the change in his position relative

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‘invisible masses’, ‘unconscious mental events’ are norms of expression. [...]. Whether all velocities can be accounted for by the assumption of invisible masses is a question of mathematics, or grammar, and is not to be settled by experience. It is settled beforehand.” —Ludwig Wittgenstein ([1932–1935] 2001, p. 16)

- 21 “But we have accumulated around the terms ‘force’ and ‘electricity’ more relations than can be completely reconciled amongst themselves. We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections that it can be answered; but by removing the contradictions existing between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions.” —Heinrich Hertz ([1894] 1899, pp. 7–8)
- 22 “Can we, without destroying the clearness of our conceptions, take the effect of inertia twice into account,—firstly as mass, secondly as force? In our laws of motion, force was a cause of motion, and was present *before* the motion. Can we, without confusing our ideas, suddenly begin to speak of forces which arise through motion, which are a consequence of motion?” —Heinrich Hertz ([1894] 1899, p. 6)
- 23 “No one has ever yet understood what centrifugal force really is. Is the force of the Earth rotating on its axis the cause of this motion or vice versa? Or are the two identical? Is such a cause, considered *per se*, a force or another motion? What is the difference between force and motion? Suppose the alterations in the planetary system to be workings of a centrifugal force, that being so, the bodies ought to be slung out on their path [tangentially], and as in fact they are not so, we must assume a centripetal force as well. What do all these words mean? [...]. ‘Can we speak of forces which owe their origin to motion?’ Certainly not; but can we get rid of *primary* notions that are *inborn* in the Western spirit though indefinable?” —Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §XII, p. 416)
- 24 “The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.” —Isaac Newton ([1687] 1934a, Axioms, or Laws of Motion, p. 13)
- 25 “[I]n very many cases the forces which are used in mechanics for treating physical problems are simply sleeping partners, which keep out of the business altogether when actual facts have to be represented.” —Heinrich Hertz ([1894] 1899, pp. 11–12)

to other parts of the train; but if he closed his eyes, he couldn't tell—that is, his body wouldn't be compressed and he wouldn't feel anything at all. On the other hand, the passenger is not accelerated relative to other objects on the train; but his body is compressed (and so could detect the motion of the train with his eyes closed). Insofar as a body is fixed to the train (e.g., sitting in a seat), that body does not itself move, but it is compressed. Insofar as the body moves freely (e.g., on roller skates), it moves as a unit, but it is not compressed. The effect on the passenger is *internal* to his body (compression); the effect on the steward is *translational* motion.

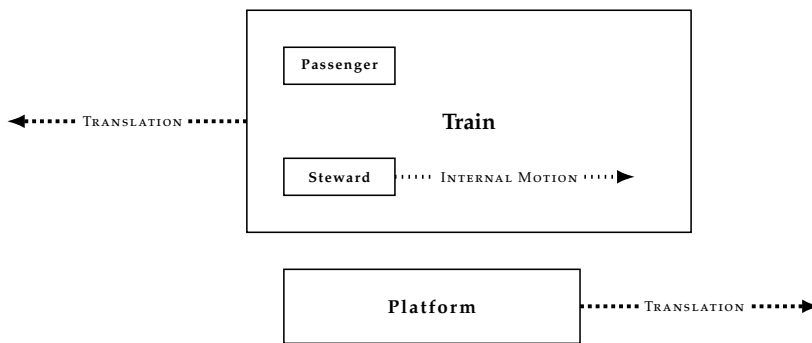


Figure 1: Inertial motion is motion inside a composite body that is itself moving in translation. Inertial forces are motion inside a composite body that is itself inside another composite body. The passenger feels a force, but the steward does not.

In the Mechanical paradigm, there is no such thing as internal motion and there is, *pace* Spengler,<sup>26</sup> no cause and effect (which contains a temporal asymmetry), so every cause for internal motion (like the compression of a spring under a weight) is a mere change in the translational motion of simple constituent

<sup>26</sup> Spengler [1918] 1926a, p. 120.

bodies.<sup>27,28</sup> The force of one body pushing on another is replaced by the force of an instantaneous and reversible collision between pairs of simple, rigid bodies (between the simple, rigid constituent parts of complex bodies). So a river cannot flow upstream, but a rock may fall to the ground or it may be thrown up in the air (and a video of each, when played in reverse, looks just like that of the other). The difference is that the model of the rock is a model of a simple body, without internal motion.<sup>29</sup>

Accordingly, in Newtonian mechanics, the existence of forces in a given model depends on the frame of reference chosen, even as one might otherwise say that a spring is compressed by the same force, independent of the frame of reference (the measurement of the magnitude of that compression being unaffected by who performs the measurement and from where). So, in the train for example, one usually describes the steward's body as experiencing no force in the frame of reference of the train platform (since the steward is stationary in that frame), but as experiencing a so-called *inertial* ('fictitious,' 'pseudo-') force in the frame of the accelerating train that produces the rearward acceleration.<sup>30</sup> And yet, isn't the platform also accelerating rearward from the perspective of the train? Why don't we say that the platform is being pulled backwards by an inertial force just like the steward? The only difference is that the latter is inside the train and the former is outside. Similarly, the difference between the passenger's body

27 "The concept of pure motion detached from any other change stands out in full clarity only in the study of rigid bodies, where indeed we have a perfectly immutable structure in which nothing changes save spatial position. Now there are in nature no perfectly rigid bodies but certainly solid ones that are subject to only imperceptible changes of shape during motion. As for changes in shape in liquids and gases, these one tries, without straining the facts, to reduce to the motions of their smallest constituent parts. Indeed to the eye they resemble changes of shape of a sandheap, which consists of individual perceptible grains. Nevertheless, in the case of actual fluids there is something hypothetical about the assumption that there too each individual particle is identifiable at all times, for experience shows that we are given only the constancy of total mass and weight." —Ludwig Boltzmann (1974e, p. 142)

28 "On this blueprint of nature that is assumed to be already known, the following conditions—among others—are entered. Motion means change of place. No motion or direction of motion is preferred to another. Every place is like every other. No point of time is superior to any other. Every force is determined only by—is only—its consequences in motion, that is to say, by the magnitude of change of place in the unit of time. Every event must be read into this blueprint of nature." —Martin Heidegger ([1938] 1976, p. 344)

29 "[O]nly such forces exist as can be represented as a sum of mutual actions between infinitely small elements of matter." —Heinrich Hertz ([1894] 1899, p. 10)

30 "[I]n an accelerated system *apparent forces, inertial forces*, act besides the true forces. A body on which no true forces act is yet subject to these inertial forces, and its motion is therefore in general neither uniform nor rectilinear. For example, a vehicle when being set into motion or stopped is such an accelerated system. Railway journeys have made everyone familiar with the jerk due to the train starting or stopping, and this is nothing other than the inertial force of which we have spoken." —Max Born ([1920] 1922, Ch. 3, §8)

being pulled backward into the seat (on one hand) and the train pushing his body forward (on the other) is whether the passenger is or is not considered to be a part of the train. It is a confusion between the translational and internal motion of the steward and the passenger each that is responsible for the identification of these ‘fictitious forces,’ which exist as motion only on the inside of a forward-accelerating body (the passenger), which itself is inside a *second* forward-accelerating body (the train). Indeed, the inertial forces exist only when describing the translation of a body that is *inside* the frame of reference, so to speak.

One ascribes the existence of ‘inertial forces’ to the ‘acceleration of the frame of reference’: when a frame of reference is not moving at a constant velocity, then in that frame of reference there are now supposed to be these (inertial) forces which do not exist at all in so-called ‘inertial’ frames.<sup>31</sup> But what does it mean for a frame of reference to be moving in the first place, except in the context of absolute space? What does it even mean to be ‘in the frame of the train’? Which part of the inside of the train are you taking the measurement from?

The description of ‘non-inertial reference frames’ as ‘invalid’<sup>32</sup> shows us a limitation of Mechanical pictures of the world. If inertial forces are the translation of the parts of a moving composite body, then an ‘inertial frame of reference’ (where there are no inertial forces) corresponds to a perspective in which that body is a simple body, which indeed is the only case wherein its translation can be measured. The internal motion of a ‘non-inertial frame’ (e.g., compression of the passenger in the accelerating train) is not defined relative to any point of reference, but is rather measured identically no matter where the measurement is performed from. Accordingly, when one says, for example, ‘in the [non-inertial] frame in which the rotating object is at rest,’ the associated shift in perspective is not equivalent to any kind of coordinate transformation (a change in position, velocity, acceleration, etc. of the observer *qua* point in space, say, to somewhere on the axis of rotation); it is rather a shift from a model of that rotation as internal motion of a composite body to a model of the

<sup>31</sup> R. P. Feynman, Leighton, and Sands [1963] 2010, Ch. 12-5.

<sup>32</sup> Møller [1952] 1955, p. 4.

(circular) translation of the parts of that body as simple bodies themselves.

*Observer and the  
Observed*

How do we describe Mechanical measurements as objectively verifiable, even as measurements of relative motion are understood to be ‘observer-dependent’?<sup>33,34</sup> There is a confusion of the observer and the frame of reference:<sup>35</sup> if I should measure the speed of a train using a laser rangefinder attached to a moving car, then the frame of reference for that measurement is the car, but *I* am still the one making the observation. It’s the statement of relative motion that is the objective one, as the identity of the observer plays no part in such a measurement.<sup>36,37</sup>

Any interaction between the observer and the system in question is considered to be a ‘disturbance’<sup>38,39</sup>—when I examine the inner workings of a machine, I open up the box and look inside; I don’t stick my hand in and feel around, for my body is a dynamical body like any other<sup>40</sup> and it would then change the system that I’m trying to observe (by getting stuck in the gears, say). The observer—and the observer’s body in particular—would then be an element in the system being observed, and it would need to be included in any model thereof, as one might say that there is an important element missing from a description of a scene,

33 “I use the word ‘observer’ in the sense in which it is conventionally used in Galilean relativity when we say that an object has a velocity ‘with respect to a certain observer’. The observer can be any physical object having a definite state of motion.” —Carlo Rovelli (1996, p. 3)

34 “When you sit with a nice girl for two hours you think it’s only a minute, but when you sit on a hot stove for a minute you think it’s two hours. That’s relativity.” —Albert Einstein (“[Einstein is Found Hiding on Birthday](#)” 1929)

35 Cf. “Of necessity a motion is relative when experienced with respect to a ‘basis-body’ experienced at rest and in unity with my corporeal animate organism.” —Edmund Husserl ([1934] 1981, p. 224)

36 “The less anthropomorphic science believes itself to be, the more anthropomorphic it is.” —Oswald Spengler ([1918] 1926a, Ch. XI: Faustian and Apollonian Nature-Knowledge, §XV, p. 427)

37 The internal motion of another composite body is what is objective, as what is verifiable is that someone *else* is in pain: “The only valid frames of reference are ones where the observer isn’t accelerating” is analogous to “only I know how much pain I’m in.” (Cf. “It can’t be said of me at all (except perhaps as a joke) that I *know* I am in pain. What is it supposed to mean—except perhaps that I *am* in pain?¶ Other people cannot be said to learn of my sensations *only* from my behaviour,—for I cannot be said to learn of them. I have them.¶ The truth is: it makes sense to say about other people that they doubt whether I am in pain; but not to say it about myself.” —Ludwig Wittgenstein ([1953] 1958, §246))

38 “If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.” —Einstein, Podolsky, and Rosen (1935, p. 777)

39 “Indeed the *finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails—because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose—the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality.” —Niels Bohr (1935, p. 697)

40 “A stone, the body of a beast, the body of a man, my body, all stand on the same level.” —Ludwig Wittgenstein ([1961] 1979, §12.10.16)

such as a picture that I might draw of a room that I myself am standing in.<sup>41,42</sup>

If my goal is to find out how a pendulum moves—and not merely how a pendulum moves when *I* measure it in *this* particular way—then I have to observe it from a distance. Indeed, in Mechanics, the observer is physically removed from the system they are observing, which is a ‘closed system’ in relative motion with a fixed center of mass:<sup>43</sup> it is when two bodies are physically removed from one another that they are independent.<sup>44,45</sup> The grammar of Mechanical models in particular always allows for *external* observation, such that the observer himself is nothing but a perspective<sup>46</sup> or vantage point and unable to affect the system accordingly.<sup>47,48</sup> Our conventional conception of the observation of the natural world as comprising acts in which *I* necessarily play no role (i.e., the observer’s identity is irrelevant)<sup>49,50</sup> is based

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- 41 “If I wrote a book ‘The world as I found it’, I should also have therein to report on my body and say which members obey my will and which do not, etc. [...]” —Ludwig Wittgenstein ([1921] 1922, §5.631)
- 42 Cf. “[Q]uantum mechanics, as it is enshrined in textbooks, seems to require separate rules for how quantum objects behave when we’re not looking at them, and how they behave when they are being observed. [...]. The whole thing is preposterous. Why are observations special? What counts as an ‘observation,’ anyway? When exactly does it happen? Does it need to be performed by a person? Is consciousness somehow involved in the basic rules of reality? Together these questions are known as the ‘measurement problem’ of quantum theory.” —Sean Carroll (2019)
- 43 “A curious analogy could be based on the fact that the hugest telescope has to have an eye-piece no larger than the human eye.” —Ludwig Wittgenstein ([1977] 1980a, p. 17e)
- 44 “Tis utterly inconceivable, that inanimate brute matter, without the mediation of some immaterial being should operate upon and affect other matter without mutual contact; that distant bodies should act upon each other through a vacuum, without the intervention of something else, by and through which the action may be conveyed from one to the other.” —Richard Bentley ([1692] 1838, p. 162)
- 45 “However, if one abandons the assumption that what exists in different parts of space has its own, independent [from each other], real existence, then I simply cannot see what it is that physics is meant to describe. For what is thought to be a ‘system’ is, after all, just a convention, and I cannot see how one could divide the world objectively in such a way that one could make statements about parts of it.” —Albert Einstein (M. Born, H. Born, and Einstein [1916–1955] 1971, Remark within marginal comments, attached to letter 86)
- 46 “The subject of optics, [was] known as ‘perspective’ in the Middle Ages [...]” —James Hannam ([2009] 2011, p. 145)
- 47 Cf. “spooky action at a distance” (M. Born, H. Born, and Einstein [1916–1955] 1971, Letter 84)
- 48 “The Greek honored the body, and aimed at the perfect representation of it, because he deified nature, and strove to approach her as closely as possible. The Christian, on the contrary, despised the Body. He looked on Nature herself as partaking of the Fall, and thereby impure, alien from God. The Body, thus conceived as the perishable vehicle of the Soul, was not a fitting symbol. He did not try to express his Ideal in the Body, but beyond it.” —G. H. Lewes ([1855] 1856, p. 244)
- 49 “As soon as we perceive the objects around us we consider them in relation to ourselves—and rightfully so. For our entire fate depends upon whether they please or displease, attract or repel, benefit or harm us. This completely natural way of considering and judging things seems as easy as it is necessary. But it also makes us susceptible to a thousand errors that can shame us and embitter our lives. ¶ Those human beings undertake a much more difficult task whose desire for knowledge kindles a striving to observe the things of nature in and of themselves and in their relations to one another.” —Johann Wolfgang von Goethe ([1792] 2010, p. 19)
- 50 “‘I’ is not the name of a person, nor ‘here’ of a place, and ‘this’ is not a name. But they are connected with names. Names are explained by means of them. It is also true that it is characteristic of physics not to use these words.” —Ludwig Wittgenstein ([1953] 1958, §410)

on our conception specifically of the observation of Mechanical systems and the relative translational motion of simple bodies.<sup>51</sup>

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<sup>51</sup> "What has been missing [in History], till now, is *detachment* from the objects considered [. . .]. In respect of Nature [as opposed to History], this detachment has long ago been attained, though of course it was relatively easy of attainment, since the physicist can obviously systematize the mechanical-causal picture of his world as impersonally as though he himself did not exist in it." —Oswald Spengler ([1918] 1926a, Ch. III: The Problem of World-History, I. Physiognomic and Systematic, §1, p. 93)



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## GRAVITY AND INFINITE SPACE

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*Once again, therefore, there was an act like the act of Copernicus to be accomplished, an act of emancipation from the evident present in the name of infinity. This the Western soul achieved in the domain of Nature long ago, when it passed from the Ptolemaic world-system to that which is alone valid for it to-day, and treats the position of the observer on one particular planet as accidental instead of normative.*

—Oswald Spengler<sup>1</sup>

In the development of the General Theory of Relativity, Albert Einstein broke with the Newtonian tradition and stopped referring to gravity as a force, preferring to describe it instead as the very shape and form of space-time.<sup>2</sup> Why did he do that? What is it like being pulled by gravity, as compared with being pulled by an electric charge, say? What is the grammar of gravitation?

*Motion and  
Gravity*

Einstein's theory is based on the Equivalence Principle, which says that a body falling under gravity moves the same way no matter how heavy it may be, and which Galileo proved thus: Take two cannonballs, one heavier than the other, and then tie them together with a string and drop them from some height. Were the heavier cannonball to fall faster than the lighter one, then the lighter body would retard the motion of the heavier, and the composite body would fall slower than the heavier cannonball alone. But the composite body, as a unit, is even heavier than the heavy cannonball alone, and so it should fall faster. By contradiction, the two bodies must therefore fall at the same

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<sup>1</sup> Spengler [1918] 1926a, Ch. III: The Problem of World-History, I. Physiognomic and Systematic, §1, p. 94

<sup>2</sup> Wald 1984, p. 9.

rate.<sup>3</sup> The grammar of gravitation, specifically, lets one describe the same falling object as a single object or as a set of independent smaller objects, no matter how those divisions are made.<sup>4</sup> The same argument does not apply to the electrostatic force, because if an electrically charged cannonball were split in two while ‘falling’ in an electrostatic potential, then those two parts would not continue to fall adjacent to each other, but rather would repel each other strongly.

Einstein stated the Equivalence Principle in the following, more general terms: “[T]he acceleration imparted to a body by a gravitational field is independent of the nature of the body.”<sup>5</sup> That is, every atom of a falling object is moved the same way, and the mutual relation of all parts of any composite body remains unaltered. Each body is accelerated as a unit—as a simple whole—and every body moves the same in a given field. Employing the terminology established above, we may say that *gravity describes only translational motion and no internal motion*. Accordingly, a person floating out in a gravitational field doesn’t feel any forces. Einstein realized this was the case: that an object in free fall isn’t compressed under its own weight, and that a person enclosed in an elevator is therefore unable to say whether he is floating in outer space far away from everything or accelerating rapidly toward the Earth and about to crash into the ground.<sup>6</sup>

In Relativity, one describes such a frame of reference—where there is no internal motion, and no force to be felt—as ‘locally inertial.’<sup>7</sup> However, the word ‘local’ here does not refer properly to what is a *finite* distance away, as one might think, but rather to what is *zero* distance away. So, in Einstein’s Elevator,<sup>8</sup> one distinguishes between objects that are *inside* the falling body

3 Galilei 1638, *Giornata Prima*, pp. 73–77.

4 “For my part I believe that gravity is nothing but a certain natural desire [...] to gather as a unity and a whole by combining in the form of a globe.” —Nicolaus Copernicus ([1543] 1978, Book 1, Ch. 9)

5 Einstein [1921] 1923, p. 63

6 “When I was busy (in 1907) writing a summary of my work on the theory of special relativity [...] I also had to try to modify the Newtonian theory of gravitation such as to fit its laws into the theory [...]. At that moment I got the happiest thought of my life [...] Because for an observer in free-fall from the roof of a house there is during the fall—at least in his immediate vicinity—no gravitational field. That is, if the observer lets go of any bodies, they remain relative to him, in a state of rest or uniform motion, independent of their particular chemical or physical nature.” —Albert Einstein ([1920] 2002, II. The theory of General Relativity)

7 “Therefore we formulate the equivalence principle as the statement that *at every space-time point in an arbitrary gravitational field it is possible to choose a ‘locally inertial coordinate system’ such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation.*” —Steven Weinberg (1972, p. 68)

8 Einstein [1917] 1920, Ch. 20, p. 78.

(‘local’) and those that are *outside* the elevator (‘non-local’; say, a nearby aeronaut that it flies past).

What about the ‘force of gravity’ acting on a person that is standing on the surface of the Earth? As with the passenger seated in an accelerating train, this force is measurable as a change in the relations of the parts of that person’s body (e.g., the compression of their legs) and the parts of the Earth (e.g., the compression of the ground),<sup>9</sup> rather than as a change in the relation of the person as a whole to the Earth as a whole. Indeed, a body might experience the same force in an upward-accelerating elevator out in space. So, one might say that gravity doesn’t *pull* rocks toward the Earth any more than gravity *pushes* balloons upward. (How would Newton have seen gravity in the falling of an apple if he had lived underwater?) I cannot experience ‘zero-g’ in a swimming pool only insofar as I have internal organs; but those internal organs are another matter entirely. (If I drop a hammer inside of an underwater habitat, then the hammer will always fall downward; but the hammer isn’t underwater—the chamber is—and the hammer will fall to the floor and not to the seabed. I am not in outer space right now; only the Earth is.) The *force* of gravity is a *buoyant* force.<sup>10</sup> The motion of a body in a fluid such as our atmosphere may be described as a rearrangement of the internal elements of the Earth *qua* complex body. In an atmosphere, it is the density of a body that determines the rate at which it falls, and a lighter body (e.g., a balloon) may not even fall at all; precisely in outer space will one not feel any gravitational force, for it is precisely there that one moves only inertially, as a simple body.

This raises the question, what does it mean to travel through ‘outer space,’ and how is that different from, say, running through a field of grass on Earth? I can talk about how fast I’m moving through a field by counting my strides. I wouldn’t call that ‘my speed relative to the field,’ however. Which part of the field? The field is not a body, and my distance to the field isn’t changing.<sup>11</sup>

*Motion in Outer  
Space*

<sup>9</sup> “Thus the earth has deformation and continual inner motion, etc. But how can it move as a ‘whole,’ how is that conceivable?” —Edmund Husserl ([1934] 1981, p. 225)

<sup>10</sup> “Before Einstein, we thought that what we felt all day, every day, was gravity pulling us down. Einstein realized we were mistaken. What we feel is the floor pushing up on us.” —Lee Smolin (2013, p. 68)

<sup>11</sup> “The earth does not move—perhaps I may even say that it is at rest. But that can only mean that each earth-piece, which I or someone else separates off or is broken off by itself and which is at

Imagine that you're in the void of space: there are no longer any strides for you to count, of course, nor could you measure your speed through the air with a pitot tube, say, the way you might in an atmosphere—you can only measure your speed relative to other moving bodies. A small asteroid, say 10 meters across, passes right next to you. . . you should be able to measure your speed relative to it as if it were a simple object. But what if the asteroid were 10,000,000 meters across (the size of the Earth)? You would no longer be able to say that the asteroid is passing you by—you would instead be traveling across its surface, or maybe through its atmosphere, and all nearby objects would be doing the same<sup>12</sup>—as you would not say that the Earth is passing you by while you sit in a moving train.<sup>13</sup> When the asteroid is *very* large, then one speaks of motion *within* that much larger system, and the motion has the grammar of running through the field, rather than that of floating in outer space.<sup>14</sup>

Newtonian gravitation is a picture of the universe at a large scale, depicting even the most massive bodies, along with all others, as mere points in a coordinate system.<sup>15,16</sup> In this picture, there is no motion 'within' or 'through': there is only motion

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rest or in motion, is a body. The earth as a whole whose parts—if conceived by themselves as they can be separated off, as separable—are bodies; but as a 'whole' the earth is not a body." —Edmund Husserl ([1934] 1981, p. 225) (N.B. This line of thought was so controversial that it necessitated such apologies as this by Alfred Schütz: "One of the most serious misinterpretations of Husserl's attempt at an analysis of space, for instance, would be the supposition that this philosopher ever had the intention of substituting constructions of a primitive speculation for the accomplishments of modern science and mathematics, which he knew as thoroughly as anyone." (Husserl 1940, p. 22))

<sup>12</sup> "For all the earth is the same earth—on it, in it, over it, the same bodies hold sway. 'On it,' etc., the same organismal subjects, subjects of animate organisms, which, in an altered sense, are the bodies for all. For all of us, however, the earth is the basis and not a body in the complete sense." —Edmund Husserl ([1934] 1981, p. 226)

<sup>13</sup> If the train were itself in empty space, then it wouldn't mean anything to speak of its acceleration except insofar as the steward moved along with it, under gravity, that is, for it would then have no ground to push against.

<sup>14</sup> Precisely when the cannonballs in Galileo's thought experiment are much smaller than the Earth are there no gravitational tidal forces that would differentially affect the two components of the cannonballs *qua* composite body. ("Note however, that bodies which are 'large' enough to feel the tidal forces of the gravitational field will deviate from geodesic motion." —Robert M. Wald (1984, pp. 73–74))

<sup>15</sup> "[I]t is by this argument that Newton concludes *all* gravitation to be mutual; concludes, therefore, that every particle of matter, having weight proportional (at a given place) to its mass, is also a center of gravitational force, with a strength proportional to its mass." —Howard Stein (1967, p. 180)

<sup>16</sup> [...] Leibniz and Mach suggest that if we want to get a true idea of what a point of space-time is like we should look outward at the universe, not inward [...]. The complete notion of a point of space-time in fact consists of the appearance of the entire universe as seen from that point. Copernicus did not convince people that the earth was moving by getting them to examine the earth but rather the heavens. Similarly, the reality of different points of space-time rests ultimately on the existence of different (coherently related) viewpoints of the universe as a whole. —Harvey R. Brown (2005, p. 14)

in outer space<sup>17</sup>—an infinite void dotted with points of mass<sup>18</sup>—where the motion of a falling body is the same phenomenon as that of the orbits of the planets around the Sun... where Jupiter is the same kind of object as the apple on the Earth.<sup>19,20,21</sup> And it is thus that Newton's inverse-square law of gravitation is equivalent to Gauss's Law of Gravity, which says simply that, as far as gravity is concerned, every body is nothing more than a point at its center of mass with a mass equal to that of the whole.<sup>22,23</sup>

In Newton's theory, where all bodies are point masses in outer space, there is no 'up' nor 'down'—every location in space is the same.<sup>24,25</sup> And yet, on Earth, when I drop a hammer from my hand, it falls *down*, toward the ground. So, in Aristotle, light objects rise (e.g., balloons) while heavy objects fall (e.g., rocks).<sup>26</sup> Now, a balloon flies up, toward the sky; could one say that the hammer falls 'away from the sky'? When the hammer falls to

*Ptolemy and  
Copernicus*

- 17 "[R]ectilinear, uniform motion is utterly impossible, and can take place only in a vacuum." —Alexandre Koyré ([1968] 1992, p. 34)
- 18 "But now it was Space that ruled the universe. 'World' signifies space, and the stars are hardly more than mathematical points, tiny balls in the immense, that as material no longer affect the world-feeling." —Oswald Spengler ([1918] 1926a, Ch. IX: Soul-Image and Life-Feeling, I. On the Form of the Soul, §VIII, p. 329)
- 19 "For the attractive force of all parts of the earth can affect us, neither more nor otherwise, than if it were wholly concentrated in its central point, and it were this alone that influenced our sense; similarly with the attraction of a mountain, and of every stone, &c." —Immanuel Kant ([1786] 1909, p. 184)
- 20 "He regarded every heavenly body as a mathematical point, as fixed stars indeed appear to be at a first approximation of observation. Between each two of these points there was to be a force of attraction along the line joining them and inversely proportional to the square of their distance. By conceiving a similar force to be acting between any two material particles of any body whatsoever and by applying the laws of motion obtained from observations on terrestrial bodies, he succeeded in deriving from the one law the motions of all celestial bodies, gravity, the tides and all connected phenomena." —Ludwig Boltzmann ([1899] 1974c, p. 80)
- 21 "[T]his kind of a conception of space is always, under the influence of science, becoming smaller and smaller: in the same way as science has taught us, and is still teaching us, to look upon the earth as small—yea, to look upon the entire solar system as a mere point." —Friedrich Nietzsche ([1881] 1911, Book I, §7)
- 22 "In his demonstration Maupertuis refers to a theorem from Newtonian potential theory that states that only spherical bodies exert equal forces in all directions. The only definite point of reference for the determination of distance in this case is the center of the sphere. If, now, the Creator aimed at a uniformity of action in matter, the law should apply equally for a mass point as well as for an extended sphere." —Max Jammer ([1957] 1962, p. 157)
- 23 The inverse-square relation is therefore knowable *a priori* as a logical feature of such a picture of the world. ("Textbooks commonly derive the form of the force law from the assumption of Gauss' law [...]") —Craig Callender (2005, p. 118)
- 24 "The difference between one event and another does not depend on the mere difference of the times or the places at which they occur, but only on differences in the nature, configuration or motion of the bodies concerned." —James Clerk Maxwell (1876, p. 21)
- 25 "The general validity of the principle that the universe presents the same aspect from every point (and according to a modern school of cosmologists also at every time), except for local irregularities, is accepted in modern science as a necessary condition for the repeatability of experiments, since space and time are the only parameters which, at least in principle, are beyond the control of the experimenter and cannot be reproduced at his will." —Max Jammer ([1954] 1993, p. 84)
- 26 Aristotle [351 BCE] 1837, p. 214.

the ground, or even into the deepest ocean trench, eventually it'll get there. . . to the bottom. What is the balloon's destination?<sup>27</sup> The falling hammer gets farther away from some things, but also closer to others—to other, particular parts of the Earth. If I launch a rocket upward, no matter which way it flies, it will only get farther and farther away from everything else. (It is “Up, up and away!”<sup>28</sup> but “down down to Goblin-town”<sup>29</sup>.)<sup>30</sup> How is it that when ‘everything’ falls, it all falls down, toward the Earth?<sup>31</sup> How is it that if the Earth began spinning around much more quickly, then everything would be flung away from it—as though the Earth were a giant slingshot—and yet if the Sun did the same, we would not even notice? It was such considerations that led Ptolemy to call the Earth the center of the universe:

Hence I think it is idle to seek for causes for the motion of objects toward the centre, once it has been so clearly established from the actual phenomena that the earth occupies the middle place of the universe, and that all heavy objects are carried towards the earth.  
—Ptolemy<sup>32</sup>

Ptolemy's geocentric world-view, originating in the Arabian culture<sup>33</sup> and having persisted through the Renaissance, was eventually rejected and replaced by the heliocentric paradigm of Copernicus, in which the Earth is just another body in space.<sup>34</sup> The Copernican theory, in turn, was succeeded by Newton's theory of universal gravity that treats all bodies—the Earth included—

27 “*Philothous*: Shoot up an Arrow perpendicular from the Earth; the Arrow you know, will return to your foot again. ¶ *Hylobares*: If the wind hinder not. But what does this Arrow aim at?” —Henry More ([1668] 1743, First dialogue, §XXVI)

28 Webb 1967.

29 Tolkien [1937] 1966, p. 56.

30 Cf. “Two things fill the mind with ever new and increasing admiration and awe, the more often and steadily we reflect upon them: *the starry heavens above me and the moral law within me.*” —Immanuel Kant ([1788] 1889, p. 260)

31 One says all the time, “Everything is falling down.” But what could “everything is flying up” mean? (One wants to say here, instead, “Everything is flying *away*.”)

32 Ptolemy [ca. 150] 1984, p. 43

33 “The sacred book, the Canon is, scientifically as in other respects, Arabian—the scientific canon of Ptolemy (*Almagest*), the medical of Ibn Sina (*Avicenna*), and the philosophical corpus designated ‘Aristotle,’ but so largely spurious—so also the (mostly unwritten) laws and methods of quotation [...] —Oswald Spengler ([1922] 1926b, Ch. X: The State, (A) The Problem of the Estates: Nobility and Priesthood, §IV, p. 346)

34 “The earth is [...] one of the stars in the infinite world-space. The earth is a globe-shaped body [...] earth is a huge block on which smaller bodies exist and on the basis of which they also always have become, and could have become, for us by division into pieces or by separating them off from the whole.” —Edmund Husserl ([1934] 1981, p. 222)

as mere points in absolute space.<sup>35,36,37</sup> Finally, Newton's theory was overturned by Einstein's General Relativity, wherein even locations in space cannot be distinguished<sup>38</sup>—the coordinate system itself is completely arbitrary and there are no preferred frames of reference, even as General Relativity failed to abolish the notions of absolute space<sup>39</sup> and the aether<sup>40</sup>.

Still, in the present era, after all of these developments, physicists cannot resist talking about the Copernican theory as if it were maybe *half* true.<sup>41,42,43</sup> It's not so much that what Copernicus said was *correct*, rather that his negative statement—that our unique position in the universe is not at all important—expressed an idea previously completely unrecognized (and manifested in each successive cosmological model), which is that the notion that the philosophical Subject (the observer) should have a privileged position in the natural world is naïve and provincial in

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- 35 "A material particle is a characteristic by which we associate without ambiguity a given point in space at a given time with a given point in space at any other time." —Heinrich Hertz ([1894] 1899, pp. 45–46)
- 36 "In the infinite universe of Bruno [...] all 'places' are perfectly equivalent and therefore perfectly natural for all bodies." —Alexandre Koyré ([1968] 1992, p. 9)
- 37 "All the mathematical ideas that the West found for itself or borrowed from others were automatically subjected to the form-language of the Infinitesimal—and that long before the actual Differential Calculus was discovered." —Oswald Spengler ([1918] 1926a, Ch. II: The Meaning of Numbers, §XII, p. 84)
- 38 "Aristotle is once more perfectly right. An empty space (the space of geometry) is utterly destructive of the conception of a cosmic order: in an empty space there are not only no natural places, there are no *places* at all." —Alexandre Koyré ([1968] 1992, p. 28)
- 39 "It required a severe struggle to arrive at the concept of independent and absolute space, indispensable for the development of theory. It has required no less strenuous exertions subsequently to overcome [the concept of independent and absolute space]—a process which is probably by no means as yet completed." —Albert Einstein (Jammer [1954] 1993, p. xvi)
- 40 "We may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an Aether. According to the general theory of relativity space without Aether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this Aether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it." —Albert Einstein ([1920] 1922, pp. 23–24)
- 41 "[An inertial system] is a [coördinate system] in which the laws of mechanics are valid. [...] The [coördinate system] connected with the sun resembles an inertial system more than that connected with the earth. The physical laws should be applied to Copernicus' [coördinate system] rather than to Ptolemy's." —Einstein and Infeld ([1938] 1966, pp. 209–11)
- 42 "As regards the question of whether the Earth or the heavens rotates, the difference between Copernicus and Ptolemy is indeed convention as long as we take no account of the *dynamical* discoveries that completed the Copernican revolution. However, as regards the second motion that Copernicus proposed (the annual as opposed to the diurnal motion) there is an unambiguous difference between Ptolemy and Copernicus, for Ptolemy taught that the Earth is at rest relative to the stars, i.e., the distance from the centre of the Earth to the stars does not change. Thus, if motion is defined as motion relative to the matter of the universe as a whole, Copernicus is without doubt correct about his second motion and Ptolemy simply wrong. The question of the diurnal motion remains tantalizingly open." —Julian B. Barbour ([1989] 2001, p. 694)
- 43 "The sun appears to go around the Earth, but it's the other way around—and when you get right down to it, it turns out that everything moves relative to everything else." —Lee Smolin (2013, p. xi)



its anthropocentrism and unimaginativeness. And yet, is the difference between a geocentric and heliocentric model mere convention?<sup>44</sup> Should we all stop using the word ‘down’ because we might instead be living on the surface of the Sun, where it would mean something different?<sup>45</sup>

*Infinite Space*

The rejection of geocentrism was part-and-parcel with the historical development of the Mechanical world-view.<sup>46</sup> In the examination of a Mechanical system, every observation is objective,<sup>47</sup> reproducible, and predetermined; every perspective is equally valid. I am merely one passive observer of the natural world, physically removed from my object of study,<sup>48,49</sup> and my

- 44 “[If we can develop a general relativity theory, t]he struggle, so violent in the early days of science, between the views of Ptolemy and Copernicus would then be quite meaningless. Either [coördinate system] could be used with equal justification. The two sentences, ‘the sun is at rest and the earth moves,’ or ‘the sun moves and the earth is at rest,’ would simply mean two different conventions concerning two different [coördinate systems].” —Einstein and Infeld ([1938] 1966, p. 212)
- 45 “But a still better example would be that of the application of ‘above’ and ‘below’ to the earth. Here we all have a quite clear idea of what ‘above’ and ‘below’ mean. I see well enough that I am on top; the earth is surely beneath me! (And don’t smile at this example. We are indeed all taught at school that it is stupid to talk like that. But it is much easier to bury a problem than to solve it.) And it is only reflection that shews us that in this case ‘above’ and ‘below’ cannot be used in the ordinary way. (That we might, for instance, say that the people at the antipodes are ‘below’ our part of the earth, but it must also be recognized as right for them to use the same expression about us.)” —Ludwig Wittgenstein ([1953] 1958, §351)
- 46 “But rest and motion cease to be absolute as soon as the earth becomes a world-body in the open plurality of surrounding bodies. Motion and rest necessarily become relative.” —Edmund Husserl ([1934] 1981, p. 224)
- 47 “We are here faced with an epistemological problem quite new in natural philosophy, where all description of experiences has so far been based upon the assumption, already inherent in ordinary conventions of language, that it is possible to distinguish sharply between the behaviour of objects and the means of observation. This assumption is not only fully justified by all everyday experience but even constitutes the whole basis of classical physics, which, just through the theory of relativity, has received such a wonderful completion. As soon as we are dealing, however, with phenomena like individual atomic processes which, due to their very nature, are essentially determined by the interaction between the objects in question and the measuring instruments necessary for the definition of the experimental arrangements, we are, therefore, forced to examine more closely the question of what kind of knowledge can be obtained concerning the objects.” —Niels Bohr (1958, p. 25)
- 48 “If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas of physics, one is first of all struck by the following: the concepts of physics relate to a real outside world, that is, ideas are established relating to things such as bodies, fields, etc., which claim a ‘real existence’ that is independent of the perceiving subject—ideas which, on the other hand, have been brought into as secure a relationship as possible with the sense-data. It is further characteristic of these physical objects that they are thought of as arranged in a spacetime continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects ‘are situated in different parts of space’. Unless one makes this kind of assumption about the independence of the existence (the ‘being-thus’) of objects which are far apart from one another in space—which stems in the first place from everyday thinking—physical thinking in the familiar sense would not be possible. It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind. This principle has been carried to extremes in the field theory by localising the elementary objects on which it is based and which exist independently of each other, as well as the elementary laws which have been postulated for it, in the infinitely small (four-dimensional) elements of space.” —Albert Einstein (M. Born, H. Born, and Einstein [1916–1955] 1971, From “Quantum Mechanics and Reality”, essay attached to letter 88, p. 170)
- 49 “Consider, now, Western painting as it was after Leonardo, fully conscious of its mission: How does it deal with infinite space as something *singular* which comprehends both picture and spectator as mere



body is an infinitesimal point in infinite space, like all others.<sup>50</sup> The “dead Nature of Newton”<sup>51</sup> specifically deals only in the dynamics of the relative<sup>52</sup> and objective<sup>53,54</sup> translational<sup>55</sup> motion of the featureless masses<sup>56</sup> into which bodies may be divided. Indeed, it is precisely when a model describes relationships between simple bodies that the dynamics of that model must then be found in properties of the very space separating them.<sup>57,58</sup>

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centres of gravity of a spatial dynamic?” —Oswald Spengler ([1918] 1926a, Ch. IX: Soul-Image and Life-Feeling, I. On the Form of the Soul, §VIII, p. 329)

- 50 “We, on the other hand, at bottom know only the abstract space-element of the point, which can neither be seen, nor measured, nor yet named, but represents simply a centre of reference.” —Oswald Spengler ([1918] 1926a, Ch. II: Meaning of Numbers, §XII, p. 81)
- 51 Spengler [1918] 1926a, Ch. I: Introduction, §7, p. 21.
- 52 “That this requirement of general covariance, which takes away from space and time the last vestige of physical objectivity, is a natural one, will be seen from the following reflection. All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meeting of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place at the same time. The introduction of a system of reference serves no other purpose than to facilitate the description of the totality of such coincidences.” —Albert Einstein ([1916] 2008, p. 117)
- 53 “Absolute [and infinite] space is then necessary, not as a mere idea which is to serve as a rule, for considering all motion therein as merely relative, and all motion and rest must be reduced to absolute space if the phenomenon of the same is to be transformed into a definite conception of experience (which combines all phenomena).” —Immanuel Kant ([1786] 1909, p. 239)
- 54 “Just as the invariance with respect to the transformations characterizes the objective nature of reality, the structure of reason expresses itself in the arbitrariness of admissible systems. Thus it is obviously not inherent in the nature of reality that we describe it by means of coordinates; this is the subjective form that enables our reason to carry through the description [. . .]. Kant’s assertion of the ideality of space and time has been precisely formulated only in terms of the relativity of the coordinates.” —Hans Reichenbach ([1920] 1965, p. 90)
- 55 “The circular motion is (like every non-rectilinear [motion]) a continuous change of the rectilinear, and as this is itself a continuous change of relation in respect of external space, the circular motion is a change of these external relations in space, and consequently a continuous arising of new motions.” —Immanuel Kant ([1786] 1909, p. 236)
- 56 “The division of the body into *material points*, as we have it in physics, is nothing more than analysis into *simple components*.” —Ludwig Wittgenstein ([1961] 1979, 67e)
- 57 So the general theory of relativity, in its portrait of the universe as a collection of point masses on an infinite ‘spatio-temporal manifold,’ also manifests a reification of imperceptible space that Einstein described as a vindication of the aether-theory ([1920] 1922, p. 15). (Cf. “The specifically Western symbol resides not in the reality of one or of another [geometries], but in the true plurality of equally possible geometries. It is the group of space-structures—in the abundance of which the classical system is a mere particular case—that has dissolved the last residuum of the corporeal into the pure space-feeling.” —Oswald Spengler ([1918] 1926a, Ch. V: Macrocosmos, I. The Symbolism of the World-Picture and the Problem of Space, §IV, p. 176, Footnote 1))
- 58 “If one accepts the general theory of relativity as giving a literally correct picture of the physical world, one must stand with the Newtonian tradition and regard space-time as a substance.” —Tim Maudlin (1993, p. 561)

Then all that is left to describe is the shape and form<sup>59</sup> of *infinite space itself*,<sup>60</sup> the prime symbol of Faustian culture.<sup>61</sup>

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59 "The victory over the concept of absolute space or over that of the inertial system became possible only because the concept of the material object was gradually replaced as the fundamental concept of physics by that of the field. Under the influence of the ideas of Faraday and Maxwell the notion developed that the whole of physical reality could perhaps be represented as a field whose components depend on four space-time parameters. If the laws of this field are in general covariant, that is, are not dependent on a particular choice of coördinate system, then the introduction of an independent (absolute) space is no longer necessary. That which constitutes the spatial character of reality is then simply the four-dimensionality of the field. There is then no 'empty' space, that is, there is no space without a field." —Albert Einstein (Jammer [1954] 1993, p. xvii)

60 "And so ends the metaphysical doctrine of body with *emptiness* and therefore incomprehensibility, and the reason has the same fortune in all other attempts, where it strives to reach principles of the ultimate grounds of things, inasmuch as its nature is such, that it can never comprehend anything except in so far as it is determined under given conditions; consequently, since it can neither rest at the conditioned nor can make the unconditioned comprehensible, when thirst for knowledge stimulates it, to grasp the absolute totality of all conditions, nothing remains for it but to turn back from objects, upon itself, in order that instead of the ultimate boundaries of things, it may investigate and determine the ultimate boundaries of the capacity pertaining to itself." —Immanuel Kant ([1786] 1909, pp. 244–45)

61 "The Western, Gothic, form-feeling [...] is that of an unrestrained, strong-willed far-ranging soul, and its chosen badge is pure, imperceptible, unlimited space." —Oswald Spengler ([1918] 1926a, Ch. II: The Meaning of Numbers, §XII. p. 81)

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## BIBLIOGRAPHY

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- 't Hooft, G. (2016). *The Cellular Automaton Interpretation of Quantum Mechanics*. Vol. 185. Fundamental Theories of Physics. Cham, Switzerland: Springer. ISBN: 978-3-319-41284-9. DOI: [10.1007/978-3-319-41285-6](https://doi.org/10.1007/978-3-319-41285-6). arXiv: [1405.1548](https://arxiv.org/abs/1405.1548) [quant-ph].
- Anderson, P. W. (1972). "More is Different." In: *Science* 177.4047, pp. 393–96. DOI: [10.1126/science.177.4047.393](https://doi.org/10.1126/science.177.4047.393). JSTOR: [1734697](https://www.jstor.org/stable/1734697).
- Aquinas, T. [1252–1256] (1998). "On Being and Essence." In: *Selected Writings*. Ed. by R. McInerny. London, UK: Penguin Group. Chap. 3. ARK: [ark:/13960/t4cp7n28g](https://nbn-resolving.org/urn:nbn:nl:ok:ark:/13960/t4cp7n28g).
- Aristotle [351 BCE] (1837). *Aristotelis Opera*. Ed. by I. Bekker. Vol. 2. Oxford, UK: Oxonii, e Typographeo Academico.
- Arora, S. and B. Barak (2009). *Computational Complexity: A Modern Approach*. Cambridge, UK: Cambridge University Press. ISBN: 978-0-521-42426-4. URL: <http://theory.cs.princeton.edu/complexity/>.
- Barbour, J. B. [1989] (2001). *The Discovery of Dynamics: A Study from a Machian Point of View of the Discovery and the Structure of Dynamical Theories*. New York, NY: Oxford University Press. ISBN: 0-19-513202-5.
- Bell, E. T. (1945). *The Development of Mathematics*. New York, NY: McGraw-Hill Book Company, Inc. ISBN: 9780486152288.
- Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*. 1st ed. Cambridge, UK: Cambridge University Press. ISBN: 0-521-33495-0.
- Bentley, R. [1692] (1838). "A Confutation of Atheism from the Origin and Frame of the World, Part II." In: *Sermons Preached at Boyle's Lecture*. Ed. by A. Dyce. Sermon VII. London, UK: Francis Macpherson, pp. 146–172. ARK: [ark:/13960/t6wx4799m](https://nbn-resolving.org/urn:nbn:nl:ok:ark:/13960/t6wx4799m).
- Berkeley, G. [1721] (1992). *De Motu and The Analyst*. Ed. and trans. by D. M. Jesseph. The New Synthese Historical Library Texts and Studies in the History of Philosophy 41. Dordrecht, Nether-

- lands: Springer Science+Business Media Dordrecht. ISBN: 978-94-011-2592-5. DOI: [10.1007/978-94-011-2592-5](https://doi.org/10.1007/978-94-011-2592-5).
- Black, M. (1945). "A Lend-Lease Program for Philosophy and Science." In: *The Scientific Monthly* 61.3, pp. 165–172. JSTOR: [18573](https://www.jstor.org/stable/18573).
- Bloch, F. (1976). "Heisenberg and the Early Days of Quantum Mechanics." In: *Physics Today* 29.12, pp. 23–27. DOI: [10.1063/1.3024633](https://doi.org/10.1063/1.3024633).
- Bohm, D. (1952). "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables. I." In: *Physical Review* 85.2, pp. 166–179. DOI: [10.1103/PhysRev.85.166](https://doi.org/10.1103/PhysRev.85.166).
- Bohr, N. (1913). "I. On the Constitution of Atoms and Molecules." In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 26.151, pp. 1–25. DOI: [10.1080/14786441308634955](https://doi.org/10.1080/14786441308634955).
- (1933). "Light and Life." In: *Nature* 131.3309, pp. 457–459. DOI: [10.1038/131457a0](https://doi.org/10.1038/131457a0).
- (1935). "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" In: *Physical Review* 48.8, pp. 696–702. DOI: [10.1103/PhysRev.48.696](https://doi.org/10.1103/PhysRev.48.696).
- (1958). *Atomic Physics and Human Knowledge*. New York: John Wiley & Sons.
- Boltzmann, L. [1897] (1974a). "Lectures on the Principles of Mechanics: Part I." In: *Theoretical Physics and Philosophical Problems: Selected Writings*. Ed. by B. McGuinness. Trans. by P. Foulkes. Vienna Circle Collection 5. Dordrecht, Netherlands: Springer Netherlands, pp. 223–254. ISBN: 978-94-010-2091-6. DOI: [10.1007/978-94-010-2091-6\\_17](https://doi.org/10.1007/978-94-010-2091-6_17).
- [1904] (1974b). "On Statistical Mechanics." In: *Theoretical Physics and Philosophical Problems: Selected Writings*. Ed. by B. McGuinness. Trans. by P. Foulkes. Vienna Circle Collection 5. *Populäre Schriften*, Essay 19. Address given to the Scientific Congress in St. Louis, 1904. Dordrecht, Netherlands: Springer Netherlands, pp. 159–172. ISBN: 978-94-010-2091-6. DOI: [10.1007/978-94-010-2091-6\\_12](https://doi.org/10.1007/978-94-010-2091-6_12).
- [1899] (1974c). "On the Development of the Methods of Theoretical Physics in Recent Times." In: *Theoretical Physics and Philosophical Problems: Selected Writings*. Ed. by B. McGuinness. Trans. by P. Foulkes. Vienna Circle Collection 5. *Populäre*

- Schriften*, Essay 14. Address to the meeting of natural scientists at Munich 22 September 1899. Dordrecht, Netherlands: Springer Netherlands, pp. 77–100. ISBN: 978-94-010-2091-6. DOI: [10.1007/978-94-010-2091-6\\_8](https://doi.org/10.1007/978-94-010-2091-6_8).
- [1900–1902] (1974d). “On the Principles of Mechanics.” In: *Theoretical Physics and Philosophical Problems: Selected Writings*. Ed. by B. McGuinness. Trans. by P. Foulkes. Vienna Circle Collection 5. *Populäre Schriften*, Essay 17. Dordrecht, Netherlands: Springer Netherlands, pp. 129–152. ISBN: 978-94-010-2091-6. DOI: [10.1007/978-94-010-2091-6\\_10](https://doi.org/10.1007/978-94-010-2091-6_10).
- (1974e). *Theoretical Physics and Philosophical Problems: Selected Writings*. Ed. by B. McGuinness. Trans. by P. Foulkes. Dordrecht, Netherlands: D. Reidel Publishing Company. ISBN: 90-277-0249-7. DOI: [10.1007/978-94-010-2091-6](https://doi.org/10.1007/978-94-010-2091-6).
- Born, M. [1920] (1922). *Einstein’s Theory of Relativity*. Trans. by H. L. Brose. New York, NY: E. P. Dutton & Company. ARK: [ark:/13960/t77s7sp7b](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-39660-p0077-7).
- Born, M., H. Born, and A. Einstein [1916–1955] (1971). *The Born-Einstein Letters: Correspondence Between Albert Einstein and Max and Hedwig Born from 1916 to 1955*. Trans. by I. Born. With an intro. by W. Heisenberg. Forward by Bertrand Russell. London, UK: Macmillan Press Ltd. ISBN: 0-333-11267-9.
- Box, G. E. P. (1976). “Science and Statistics.” In: *Journal of the American Statistical Association* 71.356, pp. 791–799. DOI: <https://doi.org/10.2307/2286841>.
- Brans, C. H. (1988). “Bell’s Theorem Does Not Eliminate Fully Causal Hidden Variables.” In: *International Journal of Theoretical Physics* 27.2, pp. 219–226. DOI: [10.1007/bf00670750](https://doi.org/10.1007/bf00670750).
- Brown, H. R. (2005). *Physical Relativity: Space-Time Structure from a Dynamical Perspective*. New York, NY: Oxford University Press. ISBN: 978-0-19-927583-0.
- Brown, J. R. [1991] (2011). *The Laboratory of the Mind: Thought Experiments in the Natural Sciences*. 2nd ed. London, UK: Routledge. ISBN: 0-203-84779-2.
- Brown, J. R. and Y. Fehige (2019). “Thought Experiments.” In: *The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta. Winter 2019. Metaphysics Research Lab, Stanford University. URL: <https://plato.stanford.edu/archives/win2019/entries/thought-experiment/>.

- Callender, C. (2005). "Answers in Search of a Question: 'Proofs' of the Tri-Dimensionality of Space." In: *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics* 36.1, pp. 113–136. DOI: [10.1016/j.shpsb.2004.09.002](https://doi.org/10.1016/j.shpsb.2004.09.002).
- Carroll, S. (2019). "Even Physicists Don't Understand Quantum Mechanics." In: *The New York Times*. Sunday Review: Opinion. URL: <https://www.nytimes.com/2019/09/07/opinion/sunday/quantum-physics.html>.
- Cassirer, E. [1936] (1956). *Determinism and Indeterminism in Modern Physics: Historical and Systematic Studies of the Problem of Causality*. Trans. by O. T. Benfey. New Haven, CT: Yale University Press. ARK: [ark:/13960/t6rz72069](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t6rz72069).
- Clausius, R. (1857). "XI. On the Nature of the Motion which we call heat." In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 14.91, pp. 108–127. DOI: [10.1080/14786445708642360](https://doi.org/10.1080/14786445708642360).
- Cohen, M. R. (1947). *The Meaning of Human History*. The Paul Carus Lectures, 6th Series, 1944. La Salle, IL: The Open Court Publishing Company.
- Copernicus, N. [1543] (1978). *Nicholas Copernicus on the Revolutions*. Ed. by F. Dobrzycki. Trans. by E. Rosen. Vol. 2. London, UK: The Macmillan Press Ltd. ISBN: 978-0-333-14530-2.
- Curiel, E. (2019). *Schematizing the Observer and the Epistemic Content of Theories*. arXiv: [1903.02182](https://arxiv.org/abs/1903.02182) [physics.hist-ph].
- Davies, P. (1992). *The Mind of God: Science and the Search for Ultimate Meaning*. London, UK: Simon & Schuster.
- Dawkins, R. [1986] (1996). *The Blind Watchmaker: Why the Evidence of Evolution Reveals a Universe without Design*. New York, NY: W. W. Norton & Company, Inc. ISBN: 0-393-31570-3.
- Deutsch, D. (1985). "Quantum Theory as a Universal Physical Theory." In: *International Journal of Theoretical Physics* 24.1, pp. 1–41. DOI: [10.1007/BF00670071](https://doi.org/10.1007/BF00670071).
- Dirac, P. A. M. [1930] (1958). *The Principles of Quantum Mechanics*. Fourth Edition (Revised). The International Series of Monographs on Physics 27. London, UK: Oxford University Press. ISBN: 9780198520115.
- Donadi, S. and S. Hossenfelder (2022). "Toy Model for Local and Deterministic Wave-Function Collapse." In: *Phys. Rev. A*

- 106.2, 022212. DOI: [10.1103/PhysRevA.106.022212](https://doi.org/10.1103/PhysRevA.106.022212). arXiv: [2010.01327](https://arxiv.org/abs/2010.01327) [quant-ph].
- Duffing, G. (1918). *Erzwungene Schwingungen bei veränderlicher Eigenfrequenz und ihre technische Bedeutung*. Sammlung Vieweg: Tagesfragen aus den Gebieten der Naturwissenschaften und der Technik 41/42. Braunschweig, Germany: Vieweg & Sohn.
- Eddington, A. S. (1929). *The Nature of the Physical World*. The Gifford Lectures, 1927. New York, NY: The Macmillan Company. ARK: [ark:/13960/t8mc9020p](https://n2t.net/ark:/13960/t8mc9020p).
- Einstein, A. [1917] (1920). *Relativity: The Special & The General Theory: A Popular Exposition*. Trans. by R. W. Lawson. 3rd ed. London, UK: Methuen & Co. Ltd. ARK: [ark:/13960/t7tm72h7z](https://n2t.net/ark:/13960/t7tm72h7z).
- [1920] (1922). “Ether and the Theory of Relativity: An Address Delivered on May 5th, 1920, in the University of Leyden.” In: *Sidelights on Relativity*. Trans. by G. B. Jeffery and W. Perrett. London, UK: Methuen & Co. Ltd., pp. 3–24. ARK: [ark:/13960/t29884646](https://n2t.net/ark:/13960/t29884646).
- [1921] (1923). *The Meaning of Relativity: Four Lectures Delivered at Princeton University, May, 1921*. Trans. by E. P. Adams. Aberdeen, UK: Princeton University Press. ARK: [ark:/13960/t5r78td17](https://n2t.net/ark:/13960/t5r78td17).
- [1949] (1970). “Autobiographical Notes.” In: *Albert Einstein: Philosopher-Scientist*. Ed. by P. A. Schilpp. 3rd ed. The Library of Living Philosophers VII. New York, NY: MJF Books. Chap. 1, pp. 1–94. ISBN: 1-56731-432-5.
- [1920] (2002). “Fundamental Ideas and Methods of the Theory of Relativity, Present in their Development.” In: *The Collected Papers of Albert Einstein*. Vol. 7: *The Berlin Years: Writings, 1918–1921*. Ed. by M. Janssen et al. Trans. by A. Engel and S. Engelbert. Princeton, NJ: Princeton University Press. Chap. 31. ISBN: 9780691057170. URL: <https://einsteinpapers.press.princeton.edu/vol7-doc/313>.
- [1916] (2008). “The Foundation of the General Theory of Relativity.” In: *The Principle of Relativity*. Aventura, FL: BN Publishing.
- Einstein, A. and L. Infeld [1938] (1966). *The Evolution of Physics: From Early Concepts to Relativity and Quanta*. New York, NY: Simon and Schuster. ARK: [ark:/13960/t9v12sg79](https://n2t.net/ark:/13960/t9v12sg79).

- Einstein, A., B. Podolsky, and N. Rosen (1935). "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" In: *Physical Review* 47.10. American Physical Society, pp. 777–780. DOI: [10.1103/PhysRev.47.777](https://doi.org/10.1103/PhysRev.47.777).
- "Einstein is Found Hiding on Birthday" (1929). In: *The New York Times*. Wireless to the New York Times, p. 3. URL: <https://www.nytimes.com/1929/03/15/archives/einstein-is-found-hiding-on-birthday-busy-with-gift-microscope-like.html>.
- Feynman, R. [1965] (1967). *The Character of Physical Law*. Cambridge, MA: MIT Press.
- Feynman, R. P., R. B. Leighton, and M. Sands [1963] (2010). *The Feynman Lectures on Physics*. Vol. 1: *Mainly Mechanics, Radiation, and Heat*. The New Millennium Edition. New York, NY: Basic Books. URL: [https://www.feynmanlectures.caltech.edu/I\\_toc.html](https://www.feynmanlectures.caltech.edu/I_toc.html).
- Galilei, G. (1638). *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*. Leiden, Netherlands.
- [1632] (1967). *Dialogue Concerning Two Chief World Systems—Ptolemaic & Copernican*. Trans. by S. Drake. 2nd ed. Berkeley, CA: University of California Press. ISBN: 0-520-00450-7. ARK: [ark:/13960/t0mt1f800](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-63868-p0071-9).
- Gao, Z. et al. (2010). "Quantitation of Major Human Cutaneous Bacterial and Fungal Populations." In: *Journal of Clinical Microbiology* 48.10, pp. 3575–3581. DOI: [10.1128/jcm.00597-10](https://doi.org/10.1128/jcm.00597-10).
- Gibbs, J. W. [1875–1878] (1906). "On the Equilibrium of Heterogeneous Substances." In: *The Scientific Papers of J. William Gibbs*. Vol. I: *Thermodynamics*. London, UK: Longmans, Green, and Co. Chap. III.
- Goethe, J. W. von [1792] (2010). "The Experiment as Mediator of Object and Subject." Trans. by C. Holdrege. In: *In Context* 24, pp. 19–23.
- Goldstein, S. (1969). "Fluid Mechanics in the First Half of this Century." In: *Annual Review of Fluid Mechanics* 1, pp. 1–29. DOI: [10.1146/annurev.fl.01.010169.000245](https://doi.org/10.1146/annurev.fl.01.010169.000245).
- Gowers, T., J. Barrow-Green, and I. Leader, eds. (2008). *The Princeton Companion to Mathematics*. Princeton, NJ: Princeton University Press. ISBN: 978-0-691-11880-2.



- Hannam, J. [2009] (2011). *The Genesis of Science: How the Christian Middle Ages Launched the Scientific Revolution*. Washington, DC: Regnery Publishing, Inc. ISBN: 978-1-59698-155-3.
- Hawking, S. W. (2001). *The Universe in a Nutshell*. Bantam Books. ISBN: 0-553-80202-X.
- Hawking, S. W. and G. F. R. Ellis (1973). *The Large Scale Structure of Space-Time*. Cambridge Monographs on Mathematical Physics. Cambridge, UK: Cambridge University Press. DOI: [10.1017/CB09780511524646](https://doi.org/10.1017/CB09780511524646).
- Hawking, S. W. and L. Mlodinow (2010a). *The Grand Design*. 1st ed. New York, NY: Bantam Press. ISBN: 0-553-80537-1.
- Hawking, S. W. and L. Mlodinow (2010b). "The (Elusive) Theory of Everything." In: *Scientific American* 303.4, pp. 68–71. DOI: [10.1038/scientificamerican1010-68](https://doi.org/10.1038/scientificamerican1010-68).
- Heidegger, M. [1938] (1976). "The Age of the World View." Trans. by M. Grene. In: *Boundary* 2 4.2, pp. 341–55. JSTOR: [302139](https://www.jstor.org/stable/302139).
- Helmholtz, H. von [1894] (1899). Preface. In: Hertz, H. *The Principles of Mechanics: Presented in a New Form*. Trans. by D. E. Jones and J. T. Walley. London, UK: Macmillan and Co., Ltd. ARK: [ark:/13960/t6vx09d6h](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t6vx09d6h).
- Hertz, H. [1894] (1899). *The Principles of Mechanics: Presented in a New Form*. Trans. by D. E. Jones and J. T. Walley. London, UK: Macmillan and Co., Ltd. ARK: [ark:/13960/t6vx09d6h](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t6vx09d6h).
- Husserl, E. (1940). "Notizen Zur Raumkonstitution." In: *Philosophy and Phenomenological Research* 1.1. Ed. by A. Schütz, pp. 21–37. DOI: [10.2307/2103194](https://doi.org/10.2307/2103194).
- [1934] (1981). "Foundational Investigations of the Phenomenological Origin of the Spatiality of Nature." In: *Husserl: Shorter Works*. Ed. by P. McCormick and F. A. Elliston. Trans. by F. Kersten. Notre Dame, IN: University of Notre Dame Press. Chap. 12, pp. 222–233. ISBN: 0-268-01703-4. ARK: [ark:/13960/t44r5dw9k](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t44r5dw9k).
- Huygens, C. [1629–1256] (1888–1950). *Œuvres complètes de Christian Huygens. Publiées par la Société Hollandaise des Sciences*. Vol. 21: Cosmologie. La Haye: Martinus Nijhoff. ARK: [ark:/13960/t6f197k4p](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t6f197k4p).
- Hyde, D. and D. Raffman (2018). "Sorites Paradox." In: *The Stanford Encyclopedia of Philosophy*. Ed. by E. N. Zalta. Summer 2018. Metaphysics Research Lab, Stanford University. URL: <https://plato.stanford.edu/archives/sum2018/entries/sorites-paradox/>

<https://plato.stanford.edu/archives/sum2018/entries/sorites-paradox/>.

- James, W. [1884] (1956). "The Dilemma of Determinism." In: *The Will to Believe: And Other Essays in Popular Philosophy*. New York, NY: Dover Publications. GUTENBERG EBOOK: 26659.
- Jammer, M. [1957] (1962). *Concepts of Force: A Study in the Foundations of Dynamics*. 1st ed. Harper Torchbook. New York, NY: Harper and Brothers. ISBN: 0-486-40689-X.
- [1954] (1993). *Concepts of Space: The History of Theories of Space in Physics*. Third, Enlarged Edition. New York, NY: Dover Publications, Inc. ISBN: 0-486-27119-6.
- Jaynes, E. T. (1985). "Macroscopic Prediction." In: *Complex Systems — Operational Approaches in Neurology, Physics, and Computers*. Ed. by H. Haken. Berlin, Germany: Springer-Verlag, pp. 254–269.
- (2003). *Probability Theory: The Logic of Science*. Ed. by G. L. Bretthorst. Cambridge, UK: Cambridge University Press. Chap. 10. ISBN: 0521592712.
- Kant, I. [1788] (1889). *Kant's Critique of Practical Reason and Other Works on the Theory of Ethics*. Trans. by T. K. Abbott. 4th ed. London, UK: Longmans, Green & Co. ARK: [ark:/13960/t5gb1xs8p](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t5gb1xs8p).
- [1786] (1909). *Kant's Prolegomena and Metaphysical Foundations of Natural Science*. Trans. by E. B. Bax. London, UK: George Bell and Sons.
- [1790] (1914). *Kant's Critique of Judgement*. Trans. by J. H. Bernard. 2nd ed. London, UK: Macmillan and Co., Limited. ARK: [ark:/13960/s2j5w17zxk2](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-s2j5w17zxk2).
- [1884] (1993). *Opus Postumum*. Ed. by E. Förster. Trans. by E. Förster and M. Rosen. The Cambridge Edition of the Works of Immanuel Kant. Cambridge, UK: Cambridge University Press. ISBN: 9780511625169. DOI: [10.1017/CB09780511625169](https://doi.org/10.1017/CB09780511625169).
- [1755] (2012). "Universal Natural History and Theory of the Heavens or Essay on the Constitution and the Mechanical Origin of the Whole Universe According to Newtonian Principles." In: *Natural Science*. Ed. by E. Watkins. Trans. by O. Reinhardt. New York, NY: Cambridge University Press, pp. 182–308. ISBN: 9780521363945.

- Koyré, A. (1943). "Galileo and the Scientific Revolution of the Seventeenth Century." In: *Philosophical Review* 52.4, pp. 333–348. DOI: [10.2307/2180668](https://doi.org/10.2307/2180668).
- [1968] (1992). *Metaphysics and Measurement*. Trans. by R. E. W. Maddison. Classics in the History and Philosophy of Science 11. Chemin de la Sallaz, Switzerland: Gordon and Breach Science Publishers. ISBN: 2-88124-575-7.
- Kuhn, T. S. [1964] (1977). "A Function for Thought Experiments." In: *The Essential Tension: Selected Studies in Scientific Tradition and Change*. Ed. by L. Krüger. Chicago, IL: The University of Chicago Press. Chap. 10, pp. 240–265.
- Langevin, P. (1934). *Réunion Internationale de Chimie-Physique 1933*. Vol. XVI: *La notion de corpuscules et d'atomes*. Actualités Scientifiques et Industrielles 132. Paris, France: Hermann et Cie.
- Laplace, P. S. M. de [1825] (1902). *A Philosophical Essay on Probabilities*. Trans. by F. W. Truscott and F. L. Emory. 1st ed. Translation from the sixth French edition. London, UK: Chapman & Hall, Limited. ARK: [ark:/13960/t20c4wk8w](https://nbn-resolving.org/urn:nbn:uk:103-ark:/13960/t20c4wk8w).
- Lewes, G. H. [1855] (1856). *The Life and Works of Goethe: With Sketches of His Age and Contemporaries, from Published and Unpublished Sources*. Vol. I. Boston, MA: Ticknor and Fields. ARK: [ark:/13960/t48p6cr6v](https://nbn-resolving.org/urn:nbn:uk:103-ark:/13960/t48p6cr6v).
- Lorenz, E. N. (1963). "Deterministic Nonperiodic Flow." In: *Journal of the Atmospheric Sciences* 20.2, pp. 130–141. DOI: [10.1175/1520-0469\(1963\)020<0130:dnf>2.0.co;2](https://doi.org/10.1175/1520-0469(1963)020<0130:dnf>2.0.co;2).
- M. O'C. Drury [1984] (1996). "Conversations with Wittgenstein." In: *The Danger of Words and Writings on Wittgenstein*. Ed. by D. Berman, M. Fitzgerald, and J. Hayes. Bristol, UK: Thoemmes Press, pp. 97–171. ISBN: 1-85506-490-1.
- Mach, E. [1886] (1914). *The Analysis of Sensations*. Trans. by C. M. Williams and S. Waterlow. Chicago, IL: The Open Court Publishing Company. ARK: [ark:/13960/t2g73968h](https://nbn-resolving.org/urn:nbn:uk:103-ark:/13960/t2g73968h).
- [1883] (1919). *The Science of Mechanics: A Critical and Historical Account of its Development*. Trans. by T. J. McCormack. 4th ed. Chicago, IL: The Open Court Publishing Co. ARK: [ark:/13960/t5z65mf5h](https://nbn-resolving.org/urn:nbn:uk:103-ark:/13960/t5z65mf5h).
- [1905] (1976). *Knowledge and Error: Sketches on the Psychology of Enquiry*. Ed. by B. McGuinness. Trans. by T. J. McCormack

- and P. Foulkes. Vol. 3. Vienna Circle Collection. Dordrecht, Netherlands: D. Reidel Publishing Company. ISBN: 978-90-277-0282-1. DOI: [10.1007/978-94-010-1428-1](https://doi.org/10.1007/978-94-010-1428-1).
- Maclaurin, C. [1748] (1971). *An Account of Sir Isaac Newton's Philosophical Discoveries*. Hildesheim, Germany: Georg Olms Verlag. ISBN: 3487040026.
- Mann, T. [1924] (1952). *Der Zauberberg*. Berlin, Germany: S. Fischer Verlag.
- [1924] (2005). *The Magic Mountain*. A Novel. Trans. by J. E. Woods. With an intro. by A. S. Byatt. Everyman's Library 289. New York, NY: Alfred A. Knopf.
- Maudlin, T. (1993). "Buckets of Water and Waves of Space: Why Spacetime Is Probably a Substance." In: *Philosophy of Science* 60.2, pp. 183–203. JSTOR: [188350](https://www.jstor.org/stable/188350).
- Maxwell, J. C. (1876). *Matter and Motion*. New York, NY: Society for Promoting Christian Knowledge. ARK: [ark:/13960/t1jh8s65v](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-39660-p0111-8).
- Maxwell, N. (2012). "In Praise of Natural Philosophy: A Revolution for Thought and Life." In: *Philosophia* 40.4, pp. 705–715. DOI: [10.1007/s11406-012-9376-3](https://doi.org/10.1007/s11406-012-9376-3).
- Møller, C. [1952] (1955). *Theory of Relativity*. Oxford, UK: Clarendon Press. ARK: [ark:/13960/t8v980r93](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-39660-p0111-8).
- More, H. [1668] (1743). *Divine Dialogues Containing Disquisitions & Instructions Concerning the Attributes and Providence of God: In Three Volumes*. Glasgow, UK: Robert Foulis.
- Munroe, R. (2020). XKCD: *Exa-Exabyte*. URL: <https://xkcd.com/2283/>.
- Newton, I. [1687] (1934a). *Sir Isaac Newton's Mathematical Principles*. Of Natural Philosophy and his System of the World. Vol. 1: *The Motion of Bodies*. Ed. by F. Cajori. Trans. by A. Motte and F. Cajori. Berkeley, CA: University of California Press. ISBN: 0-520-00928-2.
- [1687] (1934b). *Sir Isaac Newton's Mathematical Principles*. Of Natural Philosophy and his System of the World. Vol. 2: *The System of the World*. Ed. by F. Cajori. Trans. by A. Motte and F. Cajori. Berkeley, CA: University of California Press. ISBN: 0-520-00929-0.
- [1684] (1962). "De Gravitatione et Aequipondio Fluidorum." In: *Unpublished Scientific Papers of Isaac Newton: A Selection from*

- the Portsmouth Collection in the University Library, Cambridge.* Ed. and trans. by A. R. Hall and M. B. Hall. London, UK: Cambridge University Press. ISBN: 9780521294362.
- Nietzsche, F. [1881] (1911). *The Complete Works of Friedrich Nietzsche*. Vol. 9: *Dawn of the Day*. Trans. by J. M. Kennedy. London, UK: Foulis, T. N.
- [1894] (1996). *Human, All Too Human: A Book for Free Spirits*. Trans. by R. J. Hollingdale. Cambridge: Press Syndicate of the University of Cambridge. ISBN: 0-521-562007. ARK: [ark:/13960/t3jw9bw8m](https://nbn-resolving.org/urn:nbn:uk:2009-01-13960-t3jw9bw8m).
- Norton, J. D. (1996). "Are Thought Experiments Just What You Thought?" In: *Canadian Journal of Philosophy* 26.3, pp. 333–366. DOI: [10.1080/00455091.1996.10717457](https://doi.org/10.1080/00455091.1996.10717457). JSTOR: 40231953. URL: [http://www.pitt.edu/~jdnorton/papers/Are\\_TE\\_thought.pdf](http://www.pitt.edu/~jdnorton/papers/Are_TE_thought.pdf).
- Omnès, R. (1999). *Understanding Quantum Mechanics*. Princeton, NJ: Princeton University Press. ISBN: 0-691-00435-8.
- Oresme, N. [1377] (1968). *Le Livre du ciel et du monde*. Ed. by A. D. Menut and A. J. Denomy. Trans., with an introd., by A. D. Menut. Madison, WI: University of Wisconsin Press.
- Ovid [ca. 8] (1971). *Metamorphoses*. Books 1–8. Ed. by G. P. Goold. Trans. by F. J. Miller. 2nd ed. Vol. 1. Loeb Classical Library 42. Cambridge, MA: Harvard University Press.
- Pais, A. [1982] (2005). *Subtle is the Lord: The Science and the Life of Albert Einstein*. New York, NY: Oxford University Press.
- Philalethes, E. (1650) [= T. Vaughan]. *Anthroposophia Theomagica*. Or, A Discourse of the Nature of Man and his State after Death; Grounded on his Creator's Proto-Chimistry and Verifi'd by a Practicall Examination of Principles in the Great World. London, UK: Printed by T. W. for H. Blunden. ARK: [ark:/13960/t13n2zg24](https://nbn-resolving.org/urn:nbn:uk:2009-01-13960-t13n2zg24).
- Pinheiro, M. J. (2011). "On Newton's Third Law and its Symmetry-Breaking Effects." In: *Physica Scripta* 84.5, 055004. DOI: [10.1088/0031-8949/84/05/055004](https://doi.org/10.1088/0031-8949/84/05/055004). arXiv: [1104.5011](https://arxiv.org/abs/1104.5011) [physics.class-ph].
- Planck, M. [1897] (1903). *Treatise on Thermodynamics*. Trans. by A. Ogg. London, UK: Longmans, Green and Co. ARK: [ark:/13960/t1fj2br7r](https://nbn-resolving.org/urn:nbn:uk:2009-01-13960-t1fj2br7r).

- Ptolemy, C. [ca. 150] (1984). *Almagest*. Trans. by G. J. Toomer. London, UK: Gerald Duckworth & Company. ISBN: 0-7156-1588-2. ARK: [ark:/13960/t7xm5kg5x](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-63864-p0071-9).
- Pusey, M. F., J. Barrett, and T. Rudolph (2012). "On the Reality of the Quantum State." In: *Nature Physics* 8.6, pp. 475–478. DOI: [10.1038/nphys2309](https://doi.org/10.1038/nphys2309).
- Reichenbach, H. (1956). *The Direction of Time*. Ed. by M. Reichenbach. Los Angeles, CA: University of California Press. ISBN: 0-520-01839-7.
- [1920] (1965). *The Theory of Relativity and A Priori Knowledge*. Trans. by M. Reichenbach. Berkeley, CA: University of California Press. ARK: [ark:/13960/t0cw2b99f](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-63864-p0071-9).
- Rovelli, C. (1996). "Relational Quantum Mechanics." In: *International Journal of Theoretical Physics* 35.8, pp. 1637–1678. DOI: [10.1007/BF02302261](https://doi.org/10.1007/BF02302261). arXiv: [quant-ph/9609002](https://arxiv.org/abs/quant-ph/9609002) [quant-ph].
- (2018). *The Order of Time*. Trans. by E. Segre and S. Carnell. New York, NY: Riverhead Books. ISBN: 9780735216112.
- (2022). "Memory and Entropy." In: *Entropy* 24.8, 1022. DOI: [10.3390/e24081022](https://doi.org/10.3390/e24081022).
- Russell, B. [1919] (1920). "Mathematics and Logic." In: *Introduction to Mathematical Philosophy*. 2nd ed. London, UK: George Allen & Unwin, Ltd. Chap. 18, pp. 194–206. ARK: [ark:/13960/t8jd5dq49](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-63864-p0071-9).
- Rutherford, E. (1911). "LXXIX. The Scattering of  $\alpha$  and  $\beta$  Particles by Matter and the Structure of the Atom." In: *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* 21.125, pp. 669–688. DOI: [10.1080/14786440508637080](https://doi.org/10.1080/14786440508637080).
- Schelling, F. W. J. [1799] (2004). *First Outline of a System of the Philosophy of Nature*. Ed. by D. J. Schmidt. Trans. by K. R. Peterson. SUNY Series in Contemporary Continental Philosophy. Albany, NY: State University of New York Press. ISBN: 0-7914-6003-7.
- Schrödinger, E. [1944] (1992). *What is Life?* Cambridge, UK: Cambridge University Press. ISBN: 978-1-107-60466-7.
- Schultheiss, P. et al. (2022). "The abundance, biomass, and distribution of ants on Earth." In: *Proceedings of the National Academy of Sciences* 119.40, e2201550119. DOI: [10.1073/pnas.2201550119](https://doi.org/10.1073/pnas.2201550119).

- Shinbrot, T. et al. (1992). "Chaos in a Double Pendulum." In: *American Journal of Physics* 60.6, pp. 491–499. DOI: [10.1119/1.16860](https://doi.org/10.1119/1.16860). URL: <https://hal.archives-ouvertes.fr/hal-01403609>.
- Sivasundaram, S. and K. H. Nielsen (2016). *Surveying the Attitudes of Physicists Concerning Foundational Issues of Quantum Mechanics*. arXiv: [1612.00676](https://arxiv.org/abs/1612.00676) [physics.hist-ph].
- Smith, E. and H. J. Morowitz (2016). *The Origin and Nature of Life on Earth: The Emergence of the Fourth Geosphere*. New York, NY: Cambridge University Press. ISBN: 978-1-107-12188-1.
- Smolin, L. (2013). *Time Reborn: From the Crisis in Physics to the Future of the Universe*. New York, NY: Houghton Mifflin Harcourt. ISBN: 9780547511726.
- Snobelen, S. (2012). "The Myth of the Clockwork Universe: Newton, Newtonianism, and the Enlightenment." In: *The Persistence of the Sacred in Modern Thought*. Ed. by C. L. Firestone and N. A. Jacobs, pp. 149–184.
- Spengler, O. [1918] (1923). *Der Untergang des Abendlandes: Umriss einer Morphologie der Weltgeschichte*. Vol. I: *Gestalt und Wirklichkeit*. 33–47 Auflage. Munich, Germany: C. H. Beck'sche Verlagsbuchhandlung. GUTENBERG EBOOK: [spengler/untergal/](https://www.gutenberg.org/ebooks/50000).
- [1918] (1926a). *The Decline of the West*. Vol. I: *Form and Actuality*. Trans. by C. F. Atkinson. New York, NY: Alfred A. Knopf, Inc. ARK: [ark:/13960/t0ft9tx2h](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t0ft9tx2h).
- [1922] (1926b). *The Decline of the West*. Vol. II: *Perspectives of World History*. Trans. by C. F. Atkinson. New York, NY: Alfred A. Knopf, Inc. ARK: [ark:/13960/t8pc3wd08](https://nbn-resolving.org/urn:nbn:de:hbz:5:1-3960-t8pc3wd08).
- Stein, H. (1967). "Newtonian Space-Time." In: *Texas Quarterly* 10.3, pp. 174–200.
- Strogatz, S. H. (2018). *Nonlinear Dynamics and Chaos with Student Solutions Manual: With Applications to Physics, Biology, Chemistry, and Engineering*. 2nd ed. Boca Raton, FL: CRC Press.
- Tolkien, J. R. R. [1937] (1966). *The Hobbit, or, There And Back Again*. London, UK: George Allen & Unwin. ISBN: 0-04-823070-7.
- Wald, R. M. (1984). *General Relativity*. Chicago, IL: The University of Chicago Press. ISBN: 0-226-87033-2.



- Webb, J. (1967). *Up, Up and Away*. Single by "The 5th Dimension" from the album "Up, Up and Away". Produced by Johnny Rivers and Marc Gordon. Label: Soul City.
- Weinberg, S. (1972). *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. New York, NY: John Wiley & Sons. ISBN: 0-471-92567-5.
- (1993). *Dreams of a Final Theory*. London, UK: Hutchinson.
  - (2004). *The Quantum Theory of Fields*. Vol. 1: *Foundations*. Cambridge, UK: Cambridge University Press. ISBN: 978-0521670531. DOI: [10.1017/CB09781139644167](https://doi.org/10.1017/CB09781139644167).
- Wittgenstein, L. [1921] (1922). *Tractatus Logico-Philosophicus*. Ed. and trans. by C. K. Ogden. With an intro. by B. Russell. International Library of Psychology, Philosophy and Scientific Method. London, UK: Kegan Paul, Trench, Trubner & Co., Ltd. ARK: [ark:/13960/t5jb1839z](https://nbn-resolving.org/urn:nbn:uk:103-ark:/13960/t5jb1839z).
- [1953] (1958). *Philosophical Investigations*. Trans. by G. E. M. Anscombe. 2nd ed. Oxford, UK: Basil Blackwell. ISBN: 0-631-20569-1.
  - [1964] (1975). *Philosophical Remarks*. Ed. by R. Rhees. Trans. by R. Hargreaves and R. White. Oxford, UK: Basil Blackwell. ISBN: 0-631-19130-5.
  - [1958] (1969). *Preliminary Studies for the "Philosophical Investigations", Generally known as The Blue and Brown Books*. Trans. by G. E. M. Anscombe. 2nd ed. Oxford, UK: Basil Blackwell. ISBN: 0-631-14660-1.
  - [1961] (1979). *Notebooks 1914–1916*. Ed. by G. H. von Wright and G. E. M. Anscombe. Trans. by G. E. M. Anscombe. 2nd ed. Oxford, UK: Basil Blackwell. ISBN: 0-226-90447-4.
  - [1977] (1980a). *Culture and Value*. Ed. by G. H. von Wright. Trans. by P. Winch. Amended Second Edition with English Translation. Oxford, UK: Basil Blackwell. ISBN: 0-631-12752-6.
  - [1974] (1980b). *Philosophical Grammar*. Ed. by R. Rhees. Trans. by A. Kenny. Oxford, UK: Blackwell Publishing. ISBN: 0-631-11891-8.
  - [1948–1949] (1982). *Last Writings on the Philosophy of Psychology: Preliminary Studies for Part 2 of Philosophical Investigations*. Ed. by G. H. von Wright and H. Nyman. Trans. by C. G. Luckhardt and M. A. E. Aue. Vol. 1. Oxford, UK: Basil Blackwell. ISBN: 0-631-12895-6.



- [1932–1935] (2001). *Wittgenstein's Lectures, Cambridge, 1932–1935: From the Notes of Alice Ambrose and Margaret Macdonald*. Ed. by A. Ambrose. Amherst, NY: Prometheus Books.