Wittgenstein's Method and its Application to Natural Philosophy

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As though the exact natural sciences themselves stop at the point where an encounter with metaphysics becomes unavoidable! The fact that I know and understand very little of the famous doctrines of Einstein (except that, more or less, things have a fourth dimension—namely, time) prevents me as little as it does every other intelligent layman from seeing that in this doctrine of relativity the border-line between mathematical physics and metaphysics has become fluid. Is it still 'physics,' or what is it, when they tell us—and they are telling us today —that matter is ultimately and inmostly not material, it is just one manifestation of energy, and its smallest parts, which are neither small nor large, are, though surrounded indeed by time-spatial fields of power, themselves timeless and spaceless?

— Thomas Mann [1, pp. 223–24]

> And then, during the 18th and 19th centuries, natural philosophy died. It split into empirical science on the one hand, and philosophy on the other.
>
> — Nicholas Maxwell [2, p. 2]

The story that we tell now, in our history books and in our classrooms, is that natural philosophy failed in the manner of alchemy and astrology: ignominiously, in the shadow of the success of natural science.[1] We came to believe, collectively, that the investigation of the nature of the world must be informed by, even driven by, the scientific method, with experimental proof serving as the bedrock of our epistemology, for it is otherwise impossible to avoid the pitfalls of idle speculation and sheer quackery, much less to resolve the truth value of competing theories of the underlying mechanism of the natural world.[2] Modes of inquiry into the natural world that are not based, explicitly or implicitly, on the methods of natural science are widely viewed as anachronistic and ultimately valueless.[3]

Today, the disdain for philosophy among physicists is not only widespread, but deeply held and confidently expressed. Physicists as eminent as Steven Weinberg[4] and Stephen Hawking[5] consider philosophy of science to be completely valueless, with theoretical

[1] "In the present century we are suffering from the separation of science and philosophy which followed upon the triumph of Newtonian physics in the eighteenth century." —Yvor Leclerc [3, p. 31]

[2] "It has become a tradition among those who talk glibly about science that the romantic *Naturphilosophie* of Schelling and his followers represents the lowest degradation of science and that only by completely freeing themselves from that nightmare were modern biology and medical science able to resume their scientific progress. The incident has been used by empiricists as a moral to warn us against speculative philosophy in the natural sciences." —Morris Cohen [4, p. 208]

[3] "The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements." —Albert Einstein [5, p. 777]

[4] "Only rarely did it seem to me [that philosophy of science had] anything to do with the work of science as I knew it. … I am not alone in this; I know of *no one* who has participated actively in the advance of physics in the post-war period whose research has been significantly helped by the work of philosophers." —Steven Weinberg [6, pp. 133–134]

[5] "How can we understand the world in which we find ourselves? How does the universe behave? What is the nature of reality? Where did all this come from? Did the universe need a creator? Most of us do not spend most of our time worrying about these questions, but almost all of us worry about them some of the time.¶ Traditionally these are questions for philosophy, but philosophy is dead. Philosophy has not kept up with modern developments in science, particularly physics. Scientists have become the bearers of the torch of discovery in our quest for knowledge." —Stephen Hawking [7, p. 5]

physics having an effective monopoly in the pursuit of knowledge of the natural world. Philosophers of science themselves are generally of a similar opinion, assuming that our theoretical physics says something definitive about the fundamental nature of the physical world and actively eschewing any questioning of the deeper principles on which our physics is based.[6]

I argue that this understanding of the value of philosophy as a mode of thought is based on misunderstandings of what it is that our scientific theories actually say. The philosophical method of Ludwig Wittgenstein in particular may be employed profitably to distinguish between what can and cannot be known about the natural world *a priori*, and to address outstanding problems of natural philosophy directly.

Wittgenstein's Philosophical Method

Ludwig Wittgenstein's philosophical method is based on looking at how we use language, with the goal of carefully distinguishing between expressions that have similar forms but very different meanings. It involves performing a *conceptual* analysis (rather than a psychological or a linguistic analysis) of our language, as described in following passage from the *Philosophical Investigations*:

> We must do away with all *explanation*, and description alone must take its place. And this description gets its light, that is to say its purpose, from the philosophical problems. These are, of course, not empirical problems; they are solved, rather, by looking into the workings of our language, and that in such a way as to make us recognize those workings: *in despite of* an urge to misunderstand them. The problems are solved, not by giving new information, but by arranging what we have always known. Philosophy is a battle against the bewitchment of our intelligence by means of language. [9, §109]

[6] "Metaphysics is ontology. Ontology is the most generic study of what exists. Evidence for what exists, at least in the physical world, is provided solely by empirical research. Hence the proper object of most metaphysics is the careful analysis of our best scientific theories (and especially of fundamental physical theories) with the goal of determining what they imply about the constitution of the physical world." —Tim Maudlin [8, p. 104]

When Wittgenstein writes "language" here, he is not referring to spoken or written natural language exclusively. He is referring rather to any means of communication; to any description of the world. Notably, Wittgenstein himself recognized that this conception of philosophy was directly inspired by that of the physicists Heinrich Hertz[7] and Ludwig Boltzmann[8]. [10, p. 19e]

According to Wittgenstein, the practice of philosophy involves working to resolve conceptual confusions, and his philosophical arguments often take a particular form, characterizing concepts by examining what it does and doesn't make sense to say. For example:

> [One may ask] "How can one think what is not the case? If I think that King's College is on fire when it is not on fire, the fact of its being on fire does not exist. Then how can I think it? How can we hang a thief who doesn't exist?" Our answer could be put in this form: "I can't hang him when he doesn't exist; but I can look for him when he doesn't exist". [13, p. 31]

In other words, the philosophical problem, "How can one think what is not the case?" is resolved by distinguishing how we use the verb "to think" from, for instance, how we use the verb "to hang". As Wittgenstein remarks, "When words in our ordinary language have prima facie analogous [linguistic] grammars we are inclined to try to interpret them analogously; i.e., we try to make the analogy hold throughout." [13, p. 7] The task of the philosopher is to identify such confusions and to characterize correctly the relevant concepts, and Wittgenstein's method comprises an analysis of language just insofar as the problem itself is framed within language.

[7] "But we have accumulated around the terms 'force' and 'electricity' more relations than can be completely reconciled amongst themselves. We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections that it can be answered; but by removing the contradictions existing between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions." —Heinrich Hertz [11, pp. 7–8]

[8] "Only very slowly and gradually will all these illusions recede and I regard it as a central task of philosophy to give a clear account of the inappropriateness of this overshooting the mark on the part of our thinking habits; and further, in choosing and linking concepts and words, to aim only at the most appropriate expression of the given, irrespective of our inherited habits. Then, gradually, these tangles and contradictions must disappear." —Ludwig Boltzmann [12, p. 167]

> Yes, but then how can these explanations satisfy us?—Well, your very questions were framed in this language; they had to be expressed in this language, if there was anything to ask!
>
> And your scruples are misunderstandings.
>
> Your questions refer to words; so I have to talk about words.
>
> You say: the point isn't the word, but its meaning, and you think of the meaning as a thing of the same kind as the word, though also different from the word. Here the word, there the meaning. The money, and the cow that you can buy with it. (But contrast: money, and its use.)
>
> — Ludwig Wittgenstein [9, §120]

An Example Problem in Natural Philosophy

We can apply Wittgenstein's method not only to problems relating to thinking and existence, but also to those having to do with our concepts of physics and the natural world. Consider the following passage from Richard Feynman's famous *Lectures on Physics*, which exhibits philosophical confusion specifically regarding the nature of objects:

> What *is* a chair? Well, a chair is a certain thing over there … certain?, how certain? The atoms are evaporating from it from time to time—not many atoms, but a few—dirt falls on it and gets dissolved in the paint; so to define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt, or which atoms are paint that belongs to the chair is impossible. So the mass of a chair can be defined only approximately. In the same way, to define the mass of a single object is impossible, because there are not any single, left-alone objects in the world—every object is a mixture of a lot of things, so we can deal with it only as a series of approximations and idealizations. [14, p. 7]

Now, this statement of Feynman is not a statement of physics—it makes no reference to facts. Rather, Feynman is attempting to *do philosophy* by asserting what it does and doesn't make sense to say (about the chair, that is). In the process, Feynman is actually introducing a significant conceptual confusion, one which can be resolved using Wittgenstein's method. Consider the following remark of Wittgenstein's on a similar problem:

> We have been told by popular scientists that the floor on which we stand is not solid, as it appears to common sense, as it has been discovered that the wood consists of particles filling space so thinly that it can almost be called empty. This is liable to perplex us, for in a way of course we know that the floor is solid, or that, if it isn't solid, this may be due to the wood being rotten but not to its being composed of electrons. To say, on this latter ground, that the floor is not solid is to misuse language. [13, p. 45]

The above argument of Wittgenstein's applies perfectly well to Feynman's remarks: Feynman posits that what constitutes his "chair" is "defined only approximately"… but what does that actually mean? If I say that one measurement is an approximation of another—for instance, I might say that I can approximate the number of marbles in a jar by, say, dividing the volume of the jar by the volume of each marble, rather than counting the marbles individually—that is a valid use of the word "approximation". We understand what the word "approximation" means in this case, i.e., how we use that word.

In Feynman's use, however, the word is problematic: any possible description of the chair as "approximate" requires the denial of the existence of the very chair one is describing. Indeed, anything that the chair could be an "approximation" of, in Feynman's terminology, isn't something that can be "approximated". One cannot conclude from the fact that counting the atoms that constitute a chair isn't like counting the marbles in a jar that the chair doesn't exist. The chair simply isn't made of atoms in the same way that a collection of marbles is made of marbles. Indeed, there are many differences between atoms and very small marbles, and it is those very differences that are relevant in this case.

One is inclined to respond to this argument by explaining that the meanings of words are always in some sense ill defined. But it doesn't follow from the fact that words in general do not have formal definitions that everything is an approximation of everything else. The word "approximation", for instance, is not an approximation of anything. Feynman is using the word in a manner incompatible with every other usage of it, whereas when one uses the word to describe a manner of estimating the number of marbles in a jar, the meaning is clear.

Such considerations have largely been ignored in the recent history of physics as part of the dismissal of philosophical methods generally. Ernst Cassirer,[9] Edwin Schrödinger,[10] Lee Smolin,[11] Carlo Rovelli[12] and many others[13] have all made the same philosophical error, using the word "approximation" in a manner incompatible with its actual meaning, specifically in order to explain the disconnect between our physical theories and a metaphysical notion of 'reality'. We can avoid such errors by looking closely at how we use our language—which is what we are talking about when we become confused in the first place—precisely insofar as our confusion is conceptual rather than factual.

Models of Physics and their Grammars

In philosophy, we talk about the rules that govern how one may correctly or incorrectly use expressions in language. Wittgenstein refers to these rules as constituting the "grammar" of that expression, based on the analogy with linguistics. So one may make remarks about the form of language:

> "One cannot know the future" is a grammatical remark about the concept "to know". It means something like: "That is not knowing." [20, §188]

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[9] "In the same way it is necessary to introduce new constants, such as the dielectric constant, when the behavior of gases in the presence of electric or magnetic fields is studied. And each such introduction of a new factor brings about a closer approximation to reality." —Ernst Cassirer [15, p. 86]

[10] "Physical laws rest on atomic statistics and are therefore only approximate [...]" —Erwin Schrödinger [16, p. 10]

[11] "The key step is the selection, from the entire universe, of a subsystem to study. The key point is that this is always an approximation to a richer reality." —Lee Smolin [17, p. 39]

[12] "One after another, the characteristic features of time have proved to be approximations, mistakes determined by our perspective, just like the flatness of the Earth or the revolving of the sun. The growth of our knowledge has led to a slow disintegration of our notion of time. What we call 'time' is a complex collection of structures, of layers. Under increasing scrutiny, in ever greater depth, time has lost layers one after another, piece by piece [in the history of physics]." —Carlo Rovelli [18, p. 4]

[13] "[It is the prevailing scientific wisdom that w]e, as imperfect human observers, are responsible for the difference between past and future through the approximations we introduce in our description of nature." —Ilya Prigogine [19, p. 2]

As the above remark characterizes natural language expressions that employ the verb "to know", so it is also possible to describe the features (and limitations) of physical models—and not just particular models, but all models of a particular form. For instance, the theory of classical mechanics, we may say, is a theory that is fully symmetric under time reversal.[14] In the language of Wittgenstein, time-reversal symmetry is a grammatical feature of classical mechanics; "T-symmetric" describes the grammar of the theory.

But we are misled to think that the grammar of a successful physical theory is more than that—that it in fact represents a feature of 'reality itself'. In conventional philosophy of physics, physical theories are not mere objects of study; they are the ground-truth for our metaphysics, and we study them because we believe them to be at least roughly correct representations of the nature of reality.[15] As the physicist Robert Wald put it, we believe that general relativity makes "many remarkable statements concerning the structure of space and time and the nature of the gravitational field. After one has learned the theory, one cannot help feeling that one has gained some deep insights into how nature works." [23, p. 4] We reason that if general relativity is an extremely successful physical theory, and if general relativity describes space as being curved, then it is probably the case that *in some sense* space itself must be curved *in reality*.

This is backwards. First of all, general relativity does not say, "space is curved." General relativity says nothing other than that the results of *these* measurements of length (and duration) will look like *this*.[16] We can say that general relativity describes the curvature of space merely insofar as it is a theory in which the lengths of rigid bodies change, length being a property which one should have no problem calling "spatial". We know some that some spaces are curved (like the surface of a ball) and that general relativity is a theory which is able to describe the curvature of space-time in the

[14]"The fundamental equations of mechanics do not in the least change their form if we merely change the algebraic sign of the time variable. All purely mechanical processes can therefore occur equally well in the sense of increasing and decreasing time." —Ludwig Boltzmann [21, p. 170]

[15]"That discussions of space and time are ultimately accountable to the physics of space and time is probably beyond dispute, and is in any case […] a principle that [my work shares] with most of the philosophy of physics literature." —Robert DiSalle [22, p. 7]

[16]"Time, space, and mass in themselves are in no sense capable of being made the subjects of our experience, but only definite times, space-quantities, and masses." —Heinrich Hertz [24, p. 139]

presence of gravitational fields. It was the *philosophical* 'discovery' that lengths need not be absolute (by Lorentz [25]) that provided a conceptual foundation for Einstein's development of the special and general theories of relativity.

The two statements "space is curved" and "space in the vicinity of a massive body is curved" are superficially similar—and the former certainly seems to follow from the latter—but when I say "space in the vicinity of a massive body is curved", what I mean is that "these measurements of distance and length will look like this"; when I say "space is curved", I mean rather, "this is how you are able to use the word 'space'". *Within* the context of the theory of general relativity, it makes sense to specify how one uses the word "space"—for instance to avoid making an incorrect assumption of its flatness somewhere—but general relativity itself makes no statements about space itself.[17] It certainly doesn't say that the table I'm writing on is not flat—that would be to make the same error as saying that solid matter is not in fact solid. And, of course, general relativity itself correctly predicts the flatness of that table. When we say "space[-time] is curved", we describing our models themselves, and my description of a model is not a description of reality—the model is.[18]

Thought Experiments as Arguments

Even after the death of natural philosophy as a subject, philosophical thought has continued to play a critical role in the history of the development of theoretical physics. This philosophical thought has not, however, generally taken the form of metaphysics, but rather philosophical reasoning in the form of "thought experiments" developed by scientists working on the formulation of scientific theories (e.g., Mach, Hertz, Einstein).

A thought experiment is widely understood to be a kind of experiment "performed in the laboratory of the mind" [28, p. 1] and based on "empirical data [that are] well-known and generally ac-

[17]"I had just read Weyl's book *Space, Time and Matter*, and under its influence was proud to declare that space was simply the field of linear operations.¶ 'Nonsense,' said Heisenberg, 'space is blue and birds fly through it.'" —Felix Bloch [26, p. 27]

[18]Cf. "There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature."" —Niels Bohr [27]

cepted" [29, p. 241], useful primarily as a psychological crutch. But a thought experiment is nothing more than a logical argument in the form of a hypothetical construction. Wittgenstein wrote, "What Mach calls a thought experiment is of course not an experiment at all. At bottom it is a grammatical investigation."[19] [30, p. 52] By examining various thought experiments from the history of physics, we can see how they work to reveal the grammars of the concepts with which physical theories describe the world.

Take the proof of the Law of the Lever by Archimedes. This law states, more or less, that two weights resting on a lever will balance when the ratio of their respective distances from the fulcrum is the inverse of the ratio of their masses. Archimedes' derivation of this law has, historically, been the subject of a great deal of controversy precisely because of its lack of empirical basis. Ernst Mach in particular criticized it both for being circular and implicitly based on our "general experience" [32, pp. 514–15].

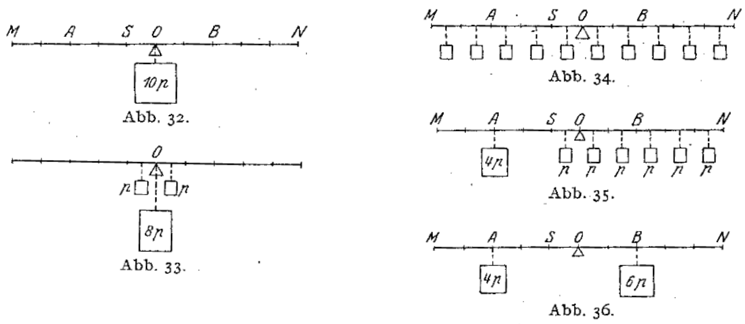

Figure 1: Hölder's reconstruction of Archimedes' proof [33, 40]

Otto Hölder, however, showed that the only assumption here, as it were, is that "displacements of equal weights are feasible (i.e., equilibrium conserving) when symmetric to any point on the configuration." [34, p. 101]. This is a principle embedded in our very notions of balance and symmetry. Indeed, it's worth thinking about cases where two identical bodies placed the same distance from the fulcrum *wouldn't* be in equilibrium, which shows the domain of applicability of the argument… the Law of the Lever doesn't apply to fire or flowing water, for instance, and that points to why we don't call those things "bodies". This means htat we can call this

[19] This position, for what it is worth, has been supported by at least one notable contemporary philosopher of science, John D. Norton, who acknowledged that "thought experiments are arguments." [31, p. 354]

argument for the Law of the Lever a "proof", but it is not a proof like one in mathematics—we are describing the properties of concepts like 'body', 'divsion', etc., rather than deriving new rules from existing ones.

A Priori Knowledge of the Natural World

Another example of a logical argument relating to physical concepts is Galileo's proof of the Weak Equivalence Principle, which follows similar reasoning and faces the same sort of criticism as Archimedes' proof. As one author put it, "The problem [in the analysis of Galileo's proof] is then to spell out exactly what would constitute a *proper unification* of bodies." [35, p. 7]

> If we had two moveables whose natural speeds were unequal, it is evident that were we to connect the slower to the faster, the latter would be partly retarded by the slower, and this would be partly speeded up by the faster….
>
> But if this is so, and if it is also true that a large stone is moved with eight degrees of speed, for example, and a smaller one of four [degrees], then joining both together, their composite will be moved with a speed less than eight degrees. But the two stones joined together make a larger stone than the first one which was moved with eight degrees of speed; therefore this greater stone moved less swiftly than the lesser one. But this is contrary to your assumption. So you see how, from the supposition that the heavier body is moved more swiftly than the less heavy, I conclude that the heavier move less swiftly. [36, pp. 66–67]

This argument shows definitively that mass alone cannot be responsible for a difference in the rate at which a body falls. Of course, it also provides the historical basis for Einstein's Equivalence Principle, which in turn has an *a priori* proof in the thought experiment of the elevator in free fall. [37, p. 21]

Galileo himself said of the Weak Equivalence Principle that he knew it to be true independent of experiment. [36, p. 62] But, one naturally asks, how can one know anything *a priori*?[20] Ironically,

[20]"At the core of the discussion sits a relatively simple epistemological challenge that is presented in a particularly powerful manner by numerous thought experiments that the history of science has to offer. They suggest that we can learn about the real world by virtue

Albert Einstein himself was one of the strongest voices asserting that it isn't possible.[21] I argue, *pace* Einstein, that while there are many things that one cannot know *a priori* (the position and momentum of this or that billiard ball, for instance), we *can* know *a priori* the features of our own models of physics. The Law of the Lever and the Equivalence Principle… these are not theoretical constructs; they are descriptions of the grammar of bodies at rest and in motion.

So, on the face of it, the following argument for the relativity of motion—also by Galileo—appears to be making an empirically verifiable prediction:

> Drop a lead ball from the top of the mast of a boat at rest, noting the place where it hits, which is close to the foot of the mast; but if the same ball is dropped from the same place when the boat is moving, it will strike that distance from the foot of the mast which the boat will have run during the time of fall of the lead, and for no other reason than that the natural movement of the ball when set free is in a straight line toward the center of the earth. [40, p. 126]

… but what kind of result would be required to disprove this principle of relativity? Isn't it the case that, because of wind resistance, no lead ball will actually fall to the bottom of the mast? You're assuming that the wind resistance is neglibile… but why would you assume that? "Because wind resistence is simply a confounding

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of merely thinking about imagined scenarios. But how can we learn about reality (if we can at all), just by thinking in such a way? Are there really thought experiments that enable us to acquire new knowledge about nature without new empirical data? If so, where does the new information come from if not from contact with the realm of investigation under consideration in an imagined scenario? Finally, how can we distinguish good from bad instances of thought experiments? These questions seem urgent with respect to scientific thought experiments, because many 'recognize them as an occasionally potent tool for increasing our understanding of nature… Historically their role is very close to the double one played by actual laboratory experiments and observations. […]'" —Stanford Encyclopedia of Philosophy [38]

[21] "The only justification for our concepts and system of concepts is that they serve to represent the complex of our experiences; beyond this they have no legitimacy. I am convinced that the philosophers have had a harmful effect upon the progress of scientific thinking in removing certain fundamental concepts from the domain of empiricism where they are under our control to the intangible heights of the *a priori*. For even if it should appear that the universe of ideas cannot be deduced from experience by logical means but is in a sense a creation of the human mind without which no science is possible, nevertheless this universe of ideas is just as little independent of the nature of our experiences as clothes are of the form of the human body. This is particularly true of our concepts of time and space which physicists have been obliged by the facts to bring down from the Olympus of the *a priori* in order to adjust them and put them in a serviceable condition." —Albert Einstein [39, p. 2]

factor: we can imagine a ship moving in a vacuum, and then the lead ball will fall straight down." How do you know? If the ball only falls straight down when there are no 'confounding factors', what purpose does your experimental verification actually serve in proving the *general* validity of the principle, when any possible experimental verification will necessarily include some of these confounding factors? How do you know which are the confounding factors and which are the fundamental properties?

In fact, Galileo's argument says nothing whatsoever about whether some or other rock when dropped from the mast of a ship will *actually* fall parallel to the mast. The thought experiment does not describe events, but rather it makes it clearer what we mean when we say, "The lead ball fell down." This thought experiment, like others, is an investigation into the grammar of motion (and, in particular, motion in a vacuum); the principle of Galilean Relativity is a description of precisely this grammar.[22] The core task of *physics*, by contrast, is to construct theories described by these principles and then to test the predictions of those theories.

Yet, this conception of the difference between philosophy and natural science poses a challenge: How do we distinguish between *good* and *bad* philosophy?[20] If we cannot rely on experimentation to verify our core physical principles, what is to stop us from constructing metaphysical castles in the sky, as indeed was common practice historically? How do we prevent ourselves from making logical mistakes in these arguments? Well, of course we can't; we can simply identify them on a case-by-case basis.

An example of such an error, and its resolution, may be found in the history of the "javelin argument" of Lucretius: Imagine that you are standing at the boundary of space and you throw a spear at it. Either the spear passes through the boundary—in which case, there is space on the other side as well—or the spear must strike something—in which case, that boundary must itself be bounded on its other side, and so there must be space too on the other side of the boundary. Therefore space must be infinite. [42, I.959–83] Now we recognize the logical mistake: Lucretius had incorrectly

[22] "Or like saying that a die must fall on one of six sides. When the possibility of a die's falling on edge is excluded, and not because it is a matter of experience that it falls only on its sides, we have a statement which no experience will refute—a statement of grammar. Whenever we say that something *must* be the case we are using a norm of expression. Hertz said that wherever something did not obey his laws there must be invisible masses to account for it." —Ludwig Wittgenstein [41, p. 16]

conflated boundlessness with infinity and so proved his thesis only for flat spaces, yet we know, philosophically, that a space *can* be curved. In short, errors in philosophical arguments are to be expected, but they are hardly fatal to the entire philosophical enterprise.[23]

Conclusion

Natural Science—and mathematical physics in particular—is a grand edifice built on particular conceptual foundations, and it is within those foundations that many of the greatest problems lie. Questions such as, "What is the nature of space?"—widely held to fall within the domain of natural science—are in fact philosophical problems, and they must be treated accordingly. We are in the habit of confusing theoretical and philosophical reasoning, and this makes it more difficult to analyze the nature of our most successful physical theories and to understand their domains of applicability and their other limitations.

It is not a coincidence that so many of the greatest historical revolutions within theoretical physics have begun with philosophical considerations about the nature of motion and space. And it is not a coincidence that the field of theoretical physics has been, in many regards, so stagnant since the early twentieth century, when philosophical considerations were given at least a greater share of the recognition and credence that they warrant.

[23] Cf. "After all, if the developments in post-Kantian mathematics and physics show anything, they show that one central Kantian formal component—the Euclidean-Newtonian picture of space and time—is clearly not *a priori* or unrevisable. The positivists were quick to draw the conclusion that *nothing* in our knowledge of nature could be *a priori* and unrevisable in Kant's sense." —Michael Friedman [43, p. 18]

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