

WITTGENSTEIN'S METHOD AND ITS APPLICATION TO NATURAL PHILOSOPHY

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As though the exact natural sciences themselves stop at the point where an encounter with metaphysics becomes unavoidable! The fact that I know and understand very little of the famous doctrines of Einstein (except that, more or less, things have a fourth dimension—namely, time) prevents me as little as it does every other intelligent layman from seeing that in this doctrine of relativity the border-line between mathematical physics and metaphysics has become fluid. Is it still 'physics,' or what is it, when they tell us—and they are telling us today—that matter is ultimately and inmosty not material, it is just one manifestation of energy, and its smallest parts, which are neither small nor large, are, though surrounded indeed by time-spatial fields of power, themselves timeless and spaceless?

— Thomas Mann (1932, pp. 223–24)

WITTGENSTEIN'S PHILOSOPHICAL METHOD

The story that we tell, in our history books and in our classrooms, is that natural philosophy failed in the manner of alchemy and astrology—ignominiously, in the shadow of the success of natural science.¹ We came to believe collectively that the investigation of the nature of the world must always be predominated by the scientific method, with experimental proof serving as the bedrock of our epistemology, and that it is otherwise impossible to avoid the pitfalls of idle speculation and sheer quackery, much less to resolve the truth value of competing theories of the underlying mechanism of the natural world.²

Modes of inquiry into the natural world that are not based, explicitly or implicitly, on the methods of natural science are thus widely viewed as anachronistic and ultimately valueless.³ The disdain for philosophy among physicists is widespread, deeply held and confidently expressed. Physicists as eminent as Steven Weinberg⁴ and Stephen Hawking⁵ consider the philosophy of science to be completely valueless. Philosophers of science themselves quite simply assume that our theoretical physics says something definitive about the fundamental nature of the physical world and

¹“In the present century we are suffering from the separation of science and philosophy which followed upon the triumph of Newtonian physics in the eighteenth century.” (Leclerc 1972, p. 31)

²“It has become a tradition among those who talk glibly about science that the romantic *Naturphilosophie* of Schelling and his followers represents the lowest degradation of science and that only by completely freeing themselves from that nightmare were modern biology and medical science able to resume their scientific progress. The incident has been used by empiricists as a moral to warn us against speculative philosophy in the natural sciences.” (Cohen 1947, p. 208)

³“The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements.” (Einstein et al. 1935, p. 777)

⁴“Only rarely did it seem to me [that philosophy of science had] anything to do with the work of science as I knew it. ... I am not alone in this; I know of *no one* who has participated actively in the advance of physics in the post-war period whose research has been significantly helped by the work of philosophers.” (Weinberg 1993, pp. 133–134)

⁵“How can we understand the world in which we find ourselves? How does the universe behave? What is the nature of reality? Where did all this come from? Did the universe need a creator? Most of us do not spend most of our time worrying about these questions, but almost all of us worry about them some of the time.¶ Traditionally these are questions for philosophy, but philosophy is dead. Philosophy has not kept up with modern developments in science, particularly physics. Scientists have become the bearers of the torch of discovery in our quest for knowledge.” (Hawking and Mlodinow 2010, p. 5)

actively eschew any questioning of the deeper principles on which that physics is based.⁶⁷

But this understanding of the value of philosophy is based on deep misunderstandings of what it is that our scientific theories actually say. The philosophical method of Ludwig Wittgenstein in particular may be used to distinguish between what can and cannot be known about the natural world *a priori*—that is, what falls and does not fall within the domain of scientific theory. This method involves performing a *conceptual* analysis of our language (rather than a psychological or a linguistic one), as described in the following passage from the *Philosophical Investigations*:

We must do away with all *explanation*, and description alone must take its place. And this description gets its light, that is to say its purpose, from the philosophical problems. These are, of course, not empirical problems; they are solved, rather, by looking into the workings of our language, and that in such a way as to make us recognize those workings: *in despite of* an urge to misunderstand them. The problems are solved, not by giving new information, but by arranging what we have always known. Philosophy is a battle against the bewitchment of our intelligence by means of language. (1958, §109)

Wittgenstein is not referring to spoken or written natural language exclusively; he is referring rather to *any description of the world*. Wittgenstein himself recognized that this conception of philosophy was directly inspired by that of the physicists Heinrich Hertz⁸

⁶⁷“That discussions of space and time are ultimately accountable to the physics of space and time is probably beyond dispute, and is in any case [...] a principle that [my work shares] with most of the philosophy of physics literature.” (DiSalle 2006, p. 7)

⁷“Metaphysics is ontology. Ontology is the most generic study of what exists. Evidence for what exists, at least in the physical world, is provided solely by empirical research. Hence the proper object of most metaphysics is the careful analysis of our best scientific theories (and especially of fundamental physical theories) with the goal of determining what they imply about the constitution of the physical world.” (Maudlin 2007, p. 104)

⁸“But we have accumulated around the terms ‘force’ and ‘electricity’ more relations than can be completely reconciled amongst themselves. We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections that it can be answered; but by removing the contradictions existing between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions.” (Hertz 1899a, pp. 7–8)

and Ludwig Boltzmann⁹. (Wittgenstein 1980, p. 19e) Accordingly, the practice of philosophy involves working to resolve conceptual confusions. Wittgenstein's philosophical arguments often take a particular form, characterizing concepts by examining what it *does* and *doesn't make sense to say*. For example:

[One may ask] "How can one think what is not the case? If I think that King's College is on fire when it is not on fire, the fact of its being on fire does not exist. Then how can I think it? How can we hang a thief who doesn't exist?" Our answer could be put in this form: "I can't hang him when he doesn't exist; but I can look for him when he doesn't exist". (1969, p. 31)

In other words, the philosophical problem, "How can one think what is not the case?" is resolved by distinguishing how we use "to think" from, for instance, how we use "to hang". As Wittgenstein remarks, "When words in our ordinary language have *prima facie* analogous [linguistic] grammars we are inclined to try to interpret them analogously; i.e., we try to make the analogy hold throughout." (1969, p. 7) The task of the philosopher is to identify such confusions and to characterize correctly the relevant concepts.

Yes, but then how can these explanations satisfy us?—Well, your very questions were framed in this language; they had to be expressed in this language, if there was anything to ask!

And your scruples are misunderstandings.

Your questions refer to words; so I have to talk about words.

You say: the point isn't the word, but its meaning, and you think of the meaning as a thing of the same kind as the word, though also different from the word. Here the word, there the meaning. The money, and the cow that you can buy with it. (But contrast: money, and its use.) (1958, §120)

⁹"Only very slowly and gradually will all these illusions recede and I regard it as a central task of philosophy to give a clear account of the inappropriateness of this overshooting the mark on the part of our thinking habits; and further, in choosing and linking concepts and words, to aim only at the most appropriate expression of the given, irrespective of our inherited habits. Then, gradually, these tangles and contradictions must disappear." (Boltzmann 1974a, p. 167)

AN EXAMPLE PROBLEM IN NATURAL PHILOSOPHY

We can apply Wittgenstein's method not only to problems relating to thinking and existence, but also to those having to do with our concepts of physics and the natural world. Consider the following passage from Richard Feynman's *Lectures on Physics*, which exhibits philosophical confusion specifically regarding the nature of *objects*:

What *is* a chair? Well, a chair is a certain thing over there ... certain?, how certain? The atoms are evaporating from it from time to time—not many atoms, but a few—dirt falls on it and gets dissolved in the paint; so to define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt, or which atoms are paint that belongs to the chair is impossible. So the mass of a chair can be defined only approximately. In the same way, to define the mass of a single object is impossible, because there are not any single, left-alone objects in the world—every object is a mixture of a lot of things, so we can deal with it only as a series of approximations and idealizations. (2010, p. 7)

Now, this statement of Feynman is not a statement of physics—it makes no reference to facts. Rather, Feynman is attempting to *do philosophy* by asserting what it does and doesn't make sense to say (about the chair, that is). In the process, Feynman introduces a significant conceptual confusion, and one that can be resolved using Wittgenstein's method. Consider Wittgenstein's remarks on a similar problem:

We have been told by popular scientists that the floor on which we stand is not solid, as it appears to common sense, as it has been discovered that the wood consists of particles filling space so thinly that it can almost be called empty. This is liable to perplex us, for in a way of course we know that the floor is solid, or that, if it isn't solid, this may be due to the wood being rotten but not to its being composed of electrons. To say, on this latter ground, that the floor is not solid is to misuse language. (1969, p. 45)

The above argument of Wittgenstein's answers that of Feynman's very well: the latter posits that what constitutes his "chair" is "defined only approximately." But what does that actually mean? If

I say that one measurement is an approximation of another—for instance, I might say that I can approximate the number of marbles in a jar by, say, dividing the volume of the jar by the volume of each marble, rather than counting the marbles individually—that is a valid use of the word “approximation”. We understand what the word “approximation” means in this case, i.e., how we use that word.

Feynman’s use, however, is problematic: any possible description of the chair as “approximate” requires the denial of the existence of the very chair one is describing. Indeed, anything that the chair could be an “approximation” of isn’t something that can be “approximated”. One cannot conclude from the fact that counting the atoms that constitute a chair isn’t like counting the marbles in a jar that the chair doesn’t exist. The chair simply isn’t made of atoms in the same way that a collection of marbles is made of marbles. Indeed, there are many differences between atoms and very small marbles, and it is those very differences that are relevant in this case.

One is inclined to respond to Wittgenstein’s point by asserting that the meanings of words are always in some sense ill defined. But it doesn’t follow from this that *everything is an approximation of everything else*. (The word “approximation”, for instance, is not an approximation of anything.) Feynman is using the word “approximation” in a manner incompatible with every other usage of it, whereas when one uses the word to describe a manner of estimating the number of marbles in a jar, the meaning is clear.

Such considerations have largely been ignored in the recent history of physics as part of the dismissal of philosophical methods generally. Ernst Cassirer,¹⁰ Edwin Schrödinger,¹¹ Lee Smolin,¹² Carlo Rovelli¹³ and many others¹⁴ have all made the same philo-

¹⁰“In the same way it is necessary to introduce new constants, such as the dielectric constant, when the behavior of gases in the presence of electric or magnetic fields is studied. And each such introduction of a new factor brings about a closer approximation to reality.” (Cassirer 1956, p. 86)

¹¹“Physical laws rest on atomic statistics and are therefore only approximate [...]” (Schrödinger 1992, p. 10)

¹²“The key step is the selection, from the entire universe, of a subsystem to study. The key point is that this is always an approximation to a richer reality.” (Smolin 2013, p. 39)

¹³“One after another, the characteristic features of time have proved to be approximations, mistakes determined by our perspective, just like the flatness of the Earth or the revolving of the sun. The growth of our knowledge has led to a slow disintegration of our notion of time. What we call ‘time’ is a complex collection of structures, of layers. Under increasing scrutiny,

sophical error, using the word “approximation” in a manner incompatible with its actual meaning, specifically in order to explain the disconnect between our physical theories and a metaphysical notion of ‘reality’. We can avoid such errors by looking closely at how we use our language—which is what we are talking about when we become confused in the first place—precisely insofar as our confusion is conceptual rather than factual.

MODELS OF PHYSICS AND THEIR GRAMMARS

In philosophy, we talk about the rules that govern how one may correctly or incorrectly use expressions in language. Wittgenstein refers to these rules as constituting the “grammar” of that expression, based on the analogy with linguistics. So one may make remarks about the form of language:

“One cannot know the future” is a grammatical remark about the concept “to know”. It means something like: “That is not knowing.” (1982, §188)

As the above passage characterizes natural-language expressions that employ “to know”, so it is also possible to describe the features (and limitations) of physical models—and not just particular models, but all models of a particular form. For instance, the theory of classical mechanics, we may say, is a theory that is fully symmetric under time reversal.¹⁵ In the language of Wittgenstein, time-reversal symmetry is a grammatical feature of classical mechanics; “T-symmetric” describes the grammar of the theory.

But we are misled to think that the grammar of a successful physical theory is more than that—that it in fact represents a feature of ‘reality itself’. In conventional philosophy of physics, physical theories are not mere objects of study; they are the ground-truth for our metaphysics, and we study them because we believe them

in ever greater depth, time has lost layers one after another, piece by piece [in the history of physics].” (Rovelli 2018, p. 4)

¹⁴“[It is the prevailing scientific wisdom that w]e, as imperfect human observers, are responsible for the difference between past and future through the approximations we introduce in our description of nature.” (Prigogine 1997, p. 2)

¹⁵“The fundamental equations of mechanics do not in the least change their form if we merely change the algebraic sign of the time variable. All purely mechanical processes can therefore occur equally well in the sense of increasing and decreasing time.” (Boltzmann 1974b, p. 170)

to be at least roughly correct representations of the nature of reality. (DiSalle 2006, p. 7) As the physicist Robert Wald put it, we believe that general relativity makes “many remarkable statements concerning the structure of space and time and the nature of the gravitational field. After one has learned the theory, one cannot help feeling that one has gained some deep insights into how nature works.” (1984, p. 4) We reason that if general relativity is an extremely successful physical theory, and if general relativity describes space as being curved, then it is probably the case that *in some sense* space itself must be curved ‘in reality’.

But general relativity does not say, “space is curved”—this is a statement about language, and true or false *a priori*. General relativity says nothing other than that the results of *these* measurements of length (and duration) will look like *this*.¹⁶ One can say that general relativity describes the curvature of space merely insofar as it is a theory in which the lengths of rigid bodies change, length being a property which one should have no problem calling “spatial”. We know that some spaces are curved (like the surface of a ball) and that general relativity is a theory which is able to describe the curvature of space-time in the presence of gravitational fields. It was the *philosophical* ‘discovery’ that lengths need not be absolute (Lorentz 1892) that provided a conceptual foundation for Einstein’s development of the special and general theories of relativity.

The two statements “space is curved” and “space in the vicinity of a massive body is curved” are superficially similar—and the former certainly seems to follow from the latter—but when I say “space in the vicinity of a massive body is curved”, what I mean is that “these measurements of distance and length will look like this”; when I say “space is curved”, I mean rather, “this is how you are able to use the word ‘space’”. *Within* the context of the theory of general relativity, it makes sense to specify how one uses the word “space”—for instance to avoid making an incorrect assumption of its flatness somewhere; but general relativity itself does not do this.¹⁷ It certainly doesn’t say that the table I’m writing on is not

¹⁶“Time, space, and mass in themselves are in no sense capable of being made the subjects of our experience, but only definite times, space-quantities, and masses.” (Hertz 1899b, p. 139)

¹⁷“I had just read Weyl’s book *Space, Time and Matter*, and under its influence was proud to declare that space was simply the field of linear operations.¶ ‘Nonsense,’ said Heisenberg, ‘space is blue and birds fly through it.’” (Bloch 1976, p. 27)

flat—that would be to make the same error as to say that solid matter is not in fact solid. And, of course, general relativity itself correctly predicts the *flatness* of that table. When we say “space[-time] is curved”, we are describing our models themselves, and my description of a model is not a description of reality—the model is.¹⁸

THOUGHT EXPERIMENTS AS ARGUMENTS

The metaphysical statements by Feynman’s concerning the nature of “objects”, Cassirer et. al. concerning “approximation”, and e.g. Wald concerning space and time—these are all *a priori*-knowable statements about language. They confuse and conflate the relationship between *descriptions of the world* (our physical theories themselves) and *descriptions of descriptions of the world* (statements about these theories). Nevertheless, more rigorous philosophical thought has often been employed very effectively by scientists working on the formulation of scientific theories (e.g., Mach, Hertz, Einstein), in particular in the form of “thought experiments”, even as the nature of these thought experiments themselves has been widely misunderstood.

A thought experiment is often described as a kind of experiment “performed in the laboratory of the mind” (Brown 2011, p. 1) and based on “empirical data [that are] well-known and generally accepted” (Kuhn 1977, p. 241), useful primarily as a psychological crutch. But a thought experiment is nothing more than a logical argument in the form of a hypothetical construction. Wittgenstein wrote, “What Mach calls a thought experiment is of course not an experiment at all. At bottom it is a grammatical investigation.”¹⁹ (1975, p. 52) By examining various thought experiments from the history of physics, we can see how they have worked to reveal the grammars of the concepts with which physical theories describe the world going all the way back to the Classical era.

¹⁸Cf. “There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature *is*. Physics concerns what we can *say* about nature.” —Niels Bohr (Petersen 1963)

¹⁹This position, for what it is worth, has been supported by at least one notable contemporary philosopher of science, John D. Norton, who acknowledged that “thought experiments are arguments.” (Norton 1996, p. 354)

Take the proof of the Law of the Lever by Archimedes. This law states, more or less, that two weights resting on a lever will balance when the ratio of their respective distances from the fulcrum is the inverse of the ratio of their masses. Archimedes' derivation of this law has, historically, been the subject of a great deal of controversy precisely because of its lack of empirical basis. Ernst Mach in particular criticized it both for being circular and implicitly based on our "general experience" (Mach 1919, pp. 514–15).

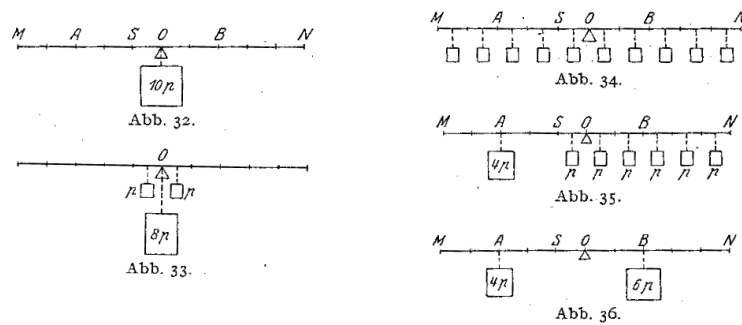


Figure 1: Hölder's reconstruction of Archimedes' proof (1924, 40)

Otto Hölder, however, showed that the only assumption here, as it were, is that "displacements of equal weights are feasible (i.e., equilibrium conserving) when symmetric to any point on the configuration." (Schlaudt 2013, p. 101). This is a principle embedded in our very notions of balance and symmetry. Indeed, it's worth thinking about cases where two identical bodies placed the same distance from the fulcrum *wouldn't* be in equilibrium, which shows the domain of applicability of the argument... the Law of the Lever doesn't apply to fire or flowing water, for instance, and that points to why we don't call those things "bodies". This means that we can call this argument for the Law of the Lever a "proof", but it is not a proof like one in mathematics—we are describing the properties of concepts like 'body', 'division', etc., rather than deriving new rules from existing ones.

***A PRIORI* KNOWLEDGE OF THE NATURAL WORLD**

Another example of a logical argument relating to physical concepts is Galileo's proof of the Weak Equivalence Principle, which follows similar reasoning and faces the same sort of criticism as Archimedes' proof. As one author put it, "The problem [in the

analysis of Galileo's proof] is then to spell out exactly what would constitute a *proper unification* of bodies." (Schrenk 2004, p. 7)

If we had two moveables whose natural speeds were unequal, it is evident that were we to connect the slower to the faster, the latter would be partly retarded by the slower, and this would be partly speeded up by the faster....

But if this is so, and if it is also true that a large stone is moved with eight degrees of speed, for example, and a smaller one of four [degrees], then joining both together, their composite will be moved with a speed less than eight degrees. But the two stones joined together make a larger stone than the first one which was moved with eight degrees of speed; therefore this greater stone moved less swiftly than the lesser one. But this is contrary to your assumption. So you see how, from the supposition that the heavier body is moved more swiftly than the less heavy, I conclude that the heavier move less swiftly. (Galilei 1954, pp. 66–67)

This argument shows definitively that mass alone cannot be responsible for a difference in the rate at which a body falls. Of course, it also provides the historical basis for Einstein's Equivalence Principle, which in turn has an *a priori* proof in the thought experiment of the elevator in free fall. (1920, p. 21)

Galileo himself said of the Weak Equivalence Principle that he knew it to be true independent of experiment. (1954, p. 62) One naturally asks, how can one know anything *a priori*?²⁰ Ironically, Albert Einstein himself was one of the strongest voices asserting that it isn't possible.²¹ While, *pace Einstein*, there are many things

²⁰"At the core of the discussion sits a relatively simple epistemological challenge that is presented in a particularly powerful manner by numerous thought experiments that the history of science has to offer. They suggest that we can learn about the real world by virtue of merely thinking about imagined scenarios. But how can we learn about reality (if we can at all), just by thinking in such a way? Are there really thought experiments that enable us to acquire new knowledge about nature without new empirical data? If so, where does the new information come from if not from contact with the realm of investigation under consideration in an imagined scenario? Finally, how can we distinguish good from bad instances of thought experiments? These questions seem urgent with respect to scientific thought experiments, because many 'recognize them as an occasionally potent tool for increasing our understanding of nature... Historically their role is very close to the double one played by actual laboratory experiments and observations. [...]" —Stanford Encyclopedia of Philosophy (Brown and Fehige 2019)

²¹"The only justification for our concepts and system of concepts is that they serve to represent the complex of our experiences; beyond this they have no legitimacy. I am convinced

that one cannot know *a priori*, (the position and momentum of this or that billiard ball, for instance), we *can* know *a priori* the features of our own models of physics. The Law of the Lever and the Equivalence Principle... these are not theoretical constructs; they are descriptions of the grammar of bodies at rest and in motion.

On the face of it, the following argument for the relativity of motion—also by Galileo—appears to be making an empirically verifiable prediction:

Drop a lead ball from the top of the mast of a boat at rest, noting the place where it hits, which is close to the foot of the mast; but if the same ball is dropped from the same place when the boat is moving, it will strike that distance from the foot of the mast which the boat will have run during the time of fall of the lead, and for no other reason than that the natural movement of the ball when set free is in a straight line toward the center of the earth. (1967, p. 126)

But what kind of result would be required to disprove this principle of relativity? Is it not the case that, because of wind resistance, no lead ball will actually fall to the bottom of the mast? You're assuming that the wind resistance is negligible... but why would you assume that? "Because wind resistance is simply a confounding factor: we can imagine a ship moving in a vacuum, and then the lead ball will fall straight down." How do you know? If the ball only falls straight down when there are no 'confounding factors', what purpose does your experimental verification actually serve in proving the *general* validity of the principle, when any possible experimental verification will necessarily include some of these confounding factors? How do you know which are the confounding factors and which are the fundamental properties?

that the philosophers have had a harmful effect upon the progress of scientific thinking in removing certain fundamental concepts from the domain of empiricism where they are under our control to the intangible heights of the *a priori*. For even if it should appear that the universe of ideas cannot be deduced from experience by logical means but is in a sense a creation of the human mind without which no science is possible, nevertheless this universe of ideas is just as little independent of the nature of our experiences as clothes are of the form of the human body. This is particularly true of our concepts of time and space which physicists have been obliged by the facts to bring down from the Olympus of the *a priori* in order to adjust them and put them in a serviceable condition." (Einstein 1923, p. 2)

In fact, Galileo's argument says nothing whatsoever about whether some or other rock when dropped from the mast of a ship will *actually* fall parallel to the mast. The thought experiment does not describe events, but rather it makes it clearer what we mean when we say, "The lead ball fell down." This thought experiment, like others, is an investigation into the grammar of motion (and, in particular, motion in a vacuum); the principle of Galilean Relativity is a description of precisely this grammar.²² The core task of *physics*, by contrast, is to construct theories described by these principles and then to test the predictions of those theories.

Yet, this conception of the difference between philosophy and natural science poses a challenge: How do we distinguish between *good* and *bad* philosophy?²⁰ If we cannot rely on experimentation to verify our core physical principles, what is to stop us from constructing metaphysical castles in the sky, as indeed was common practice historically? How do we prevent ourselves from making logical mistakes in these arguments? Well, of course we can't; we can simply identify them on a case-by-case basis.

An example of such an error, and its resolution, may be found in the history of the "javelin argument" of Lucretius: Imagine that you are standing at the boundary of space and you throw a spear at it. Either the spear passes through the boundary—in which case, there is space on the other side as well—or the spear must strike something—in which case, that boundary must itself be bounded on its other side, and so there must be space too on the other side of the boundary. Therefore space must be infinite. (Lucretius 1942, I.959–83) Now we recognize the logical mistake: Lucretius had incorrectly conflated boundlessness with infinity and so proved his thesis only for flat spaces, yet we know, philosophically, that a space *can* be curved. In short, errors in philosophical arguments are to be expected, but they are not fatal to the entire philosophical enterprise.²³

²²"Or like saying that a die must fall on one of six sides. When the possibility of a die's falling on edge is excluded, and not because it is a matter of experience that it falls only on its sides, we have a statement which no experience will refute—a statement of grammar. Whenever we say that something *must* be the case we are using a norm of expression. Hertz said that wherever something did not obey his laws there must be invisible masses to account for it." (Wittgenstein 2001, p. 16)

²³Cf. "After all, if the developments in post-Kantian mathematics and physics show anything, they show that one central Kantian formal component—the Euclidean-Newtonian picture of space and time—is clearly not *a priori* or unrevisable. The positivists were quick to draw the conclusion that *nothing* in our knowledge of nature could be *a priori* and unrevisable in Kant's sense." (Friedman 2014, p. 18)

CONCLUSION

Natural Science is a grand edifice built on particular conceptual foundations, and it is within those foundations that many of the greatest problems lie. Questions such as, “What is the nature of space?”—widely held to fall within the domain of natural science—are in fact philosophical problems, and they must be treated accordingly. When we confuse theoretical and philosophical reasoning, and this makes it more difficult to analyze the nature of our most successful physical theories and to understand their domains of applicability and their other limitations.

It is not a coincidence that so many of the greatest historical revolutions within theoretical physics have begun with philosophical considerations about the nature of motion and space. And it is not a coincidence that the field of theoretical physics has been, in many regards, so stagnant since the early twentieth century, when philosophical considerations were given at least a greater share of the recognition and credence that they warrant.

REFERENCES

- Bloch, Felix. 1976. "Heisenberg and the Early Days of Quantum Mechanics." *Physics Today* 29 (12): 23--27.
- Boltzmann, Ludwig. 1974b. *Theoretical Physics and Philosophical Problems*. Edited by Brian McGuinness. D. Reidel Publishing Company.
- Boltzmann, Ludwig. 1974a. "On Statistical Mechanics." In *Theoretical Physics and Philosophical Problems*, edited by Brian McGuinness. Springer Netherlands.
- Brown, James Robert. 2011. *The Laboratory of the Mind*. Second Edition. Routledge.
- Brown, James Robert, and Yiftach Fehige. 2019. *The Stanford Encyclopedia of Philosophy*. Winter2019 ed. Edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University. <https://plato.stanford.edu/archives/win2019/entries/thought-experiment/>.
- Cassirer, Ernst. 1956. *Determinism and Indeterminism in Modern Physics*. Yale University Press. ark.
- Cohen, Morris R. 1947. *The Meaning of Human History*. The Open Court Publishing Company.
- DiSalle, Robert. 2006. *Understanding Space-Time*. Cambridge University Press.
- Einstein, Albert. 1920. "Fundamental Ideas and Methods of the Theory of Relativity, Presented in Their Development." Princeton University Press, January 22. <https://einsteinpapers.press.princeton.edu/vol7-doc/293>.
- Einstein, Albert. 1923. *The Meaning of Relativity*. Princeton University Press. ark.
- Einstein, Albert, Boris Podolsky, and Nathan Rosen. 1935. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?." *Physical Review* 47 (10): 777--80.
- Feynman, Richard Phillips. 2010. "The Feynman Lectures on Physics." The New Millenium Edition. Vol. 1. Basic Books,. https://www.feynmanlectures.caltech.edu/I_toc.html.

- Friedman, Michael. 2014. *Foundations of Space-Time Theories: Relativistic Physics and Philosophy of Science*. Vol. 113. Princeton University Press.
- Galilei, Galileo. 1954. *Dialogues Concerning Two New Sciences*. Dover Publications.
- Galilei, Galileo. 1967. *Dialogue Concerning Two Chief World Systems—Ptolemaic & Copernican*. Second Edition. University of California Press. ark.
- Hawking, Stephen W., and L. Mlodinow. 2010. *The Grand Design*. First Edition,. Bantam Press.
- Hertz, Heinrich. 1899a. *The Principles of Mechanics*. Macmillan and Co., Ltd. ark.
- Hertz, Heinrich. 1899b. *The Principles of Mechanics, Presented in a New Form*. Macmillan and Co., Ltd.
- Hölder, Otto. 1924. “Die Mathematische Methode.” *Logisch Erkenntnistheoretische Untersuchung Im Gebiete Der Mathematik, Mechanik Und Physik*,.
- Kuhn, Thomas S. 1977. “A Function for Thought Experiments.” Chap. 10 in *The Essential Tension*, edited by Lorenz Krüger. Preprint, The University of Chicago Press,.
- Leclerc, Yvor. 1972. *The Nature of Physical Existence*. Humanities Press.
- Lorentz, Hendrik Antoon. 1892. “De Relatieve Beweging Van De Aarde En Den Aether.” *Zittingsverlag Akad. V. Wet.* 1 : 74--79.
- Lucretius. 1942. *De Rerum Natura*. Edited by William Ellery Leonard and Stanley Barney Smith. University of Wisconsin Press.
- Mach, Ernst. 1919. *The Science of Mechanics*. Fourth Edition. Open Court.
- Mann, Thomas. 1932. *Three Essays*. Martin Secker.
- Maudlin, Tim. 2007. *The Metaphysics within Physics*. Oxford University Press.

- Norton, John D. 1996. "Are Thought Experiments Just What You Thought?." *Canadian Journal of Philosophy* 26 (3): 333--66.
- Petersen, Aage. 1963. "The Philosophy of Niels Bohr." *The Bulletin of the Atomic Scientists* 19 (7).
- Prigogine, Ilya. 1997. *The End of Certainty: Time, Chaos and the New Laws of Nature*. The Free Press.
- Rovelli, Carlo. 2018. *The Order of Time*. Riverhead Books.
- Schlaudt, Oliver. 2013. "Hölder, Mach, And the Law of the Lever: A Case of Well-Founded Non-Controversy." *Philosophia Scientiæ. Travaux D'histoire Et De Philosophie Des Sciences*, nos. 17-1: 93--116.
- Schrenk, Markus A. 2004. "Galileo Versus Aristotle on Free Falling Bodies." *History of Philosophy & Logical Analysis* 7 (1): 81--89.
- Schrödinger, Erwin. 1992. *What Is Life?*. Cambridge University Press.
- Smolin, Lee. 2013. *Time Reborn*. Houghton Mifflin Harcourt.
- Wald, Robert M. 1984. *General Relativity*. The University of Chicago Press.
- Weinberg, Steven. 1993. *Dreams of a Final Theory*. Hutchinson.
- Wittgenstein, Ludwig. 1958. *Philosophical Investigations*. Second Edition. Basil Blackwell.
- Wittgenstein, Ludwig. 1969. *Preliminary Studies for the "Philosophical Investigations", Generally Known as the Blue and Brown Books*. Second Edition. Basil Blackwell.
- Wittgenstein, Ludwig. 1975. *Philosophical Remarks*. Edited by Rush Rhees. Basil Blackwell.
- Wittgenstein, Ludwig. 1980. *Culture and Value*. Amended Second Edition with English Translation. Edited by G. H. von Wright. Basil Blackwell.
- Wittgenstein, Ludwig. 1982. *Last Writings on the Philosophy of Psychology, Volume I*. Basil Blackwell.
- Wittgenstein, Ludwig. 2001. *Wittgenstein's Lectures, Cambridge, 1932-1935*. Edited by Alice Ambrose. Prometheus Books.